Requiring Minimum Sales Volume to Trigger a Commission Increase

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1. Motivation and Model Descriptions

2. Flexible Commission Margins
   - Analysis without thresholds
   - Analysis with thresholds
   - Strategic effects of thresholds

3. Fixed Commission Margins
   - Analysis without thresholds
   - Analysis with thresholds
   - Strategic effects of thresholds

4. Conclusions and Further Research
Outline

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2. Conclusions and Further Research
Why sophisticated contracts?

- Simple contracts make the chain uncoordinated:
  - Double marginalization
  - Low stocking
  - Ordering cost
Why sophisticated contracts?

- Simple contracts make the chain uncoordinated:
  - Double marginalization
  - Low stocking
  - Ordering cost

- Sophisticated contracts to achieve coordination:
  - Sale rebate (target rebate): Gallego et al. (2008)
  - Full return (buy-back): Tsay and Lovejoy (1999)
  - Revenue Sharing: Cachon and Lariviere (2005)
Sale contracts and Commission contracts

- Retailer buys the capacity from the supplier.
  - Supplier requires minimum sale volume to trigger quantity discounts.

- Provider pays broker a commission margin on each sale.
  - Provider requires minimum sale volume to trigger a commission increase.
Problem definition

We assume that the sales price of products is exogenous and fixed at $p$. 
Modelling demand

Assumptions:

- As $d$ increases, $d_0$ and $d_i$s increase proportionally.
- As $v$ increases, $d_0$ increases and $d_i$s decrease.

These assumptions are satisfied by:

- Multinomial Logit (MNL) Choice: $d_i = \frac{e^{(u_i-p)}}{e^{(u_i-p)} + e^{(u_j-p + \nu(v))}} d$
- Market Segmentation: $d_i = \beta_i(1 - \alpha(v))d$
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Formulation

Broker Problem:

\[
\max_{(s_i, \theta_i)} \pi_B = q_1 s_1 + q_2 s_2 \\
0 \leq s_i \leq \min\{b_i, d_i + \theta_i d_0\} \quad \text{for } i = 1, 2 \\
\theta_1 + \theta_2 = 1 \\
0 \leq \theta_i \quad \text{for } i = 1, 2
\]

Providers’ Best Response Problem:

\[
\max_{(q_i)} \pi_i(q_{3-i}) = (p - q_i)s_i \quad \text{for } i = 1, 2 \\
0 \leq q_i \leq p
\]
Theorem 1

- Assume $b_i > \max\{d_i, d - b_j\}$ and call it a competitive market.
- Define $m_i = \min\{b_i, d_0 + d_i\}$.
- Label the provider with higher $m$, provider 1 and the primary.

There exists a mixed-strategy Nash equilibrium such that for $q \in [0, \frac{m_1 + m_2 - d}{m_1}p]

\[
P(q_1^* \leq q) = \frac{p[m_2(d - m_2) - m_1(d - m_1)] + qm_1(d - m_1)}{(p - q)(m_1 + m_2 - d)m_1}
\]

\[
P(q_2^* \leq q) = \frac{q(d - m_2)}{(p - q)(m_1 + m_2 - d)}
\]
In equilibrium, the primary provider pays stochastically smaller commission margins.
### Competitive market

<table>
<thead>
<tr>
<th>Market situation</th>
<th>Revenue split</th>
<th>$\frac{\delta \pi_j}{\delta d}$</th>
<th>$\frac{\delta \pi_i}{\delta v}$</th>
</tr>
</thead>
</table>
| $b_1 < d_0 + d_1$| $\pi_1 = p \max[d - b_2, d_1]$  
$\pi_2 = p \frac{\min[b_2, d_0 + d_2]}{d_0 + d_1} \max[d - b_2, d_1]$  
$\pi_B = p(d - \frac{\min[b_2, d_0 + d_2] + b_1}{b_1} \max[d - b_2, d_1])$ | $> 0$                            | $\leq 0$ |
|                  |                                                                                 |                                  | $\in \mathbb{R}$               |
| $b_1 > d_0 + d_1$| $\pi_1 = p \max[d - b_2, d_1]$  
$\pi_2 = p \frac{\min[b_2, d_0 + d_2]}{d_0 + d_1} \max[d - b_2, d_1]$  
$\pi_B = p(d - \frac{\min[b_2, d_0 + d_2] + d_0 + d_1}{d_0 + d_1} \max[(d - b_2), d_1])$ | $> 0$                            | $\leq 0$ |
|                  |                                                                                 |                                  | $\in \mathbb{R}$               |

$\delta \pi_j$ and $\delta \pi_i$ denote the change in profit for different market situations. The conditions for $b_1$ and $d_0 + d_1$ determine the validity of each case.

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Requiring Minimum Sales Volume for a Commission Increase
Figure 1: Revenues when demand is not loyal

Figure: Flexible margins without thresholds
Assumption of $w = 0$ is w.l.o.g.

Corresponding to any non-negative and nondecreasing commission margin, there exists a simple commission margin function with one breakpoint starting from 0 that results in the same amount of sale and the same commission payment.
Formulation

Broker Problem:

$$\max_{(s_i, \theta_i, k_i, \Delta_i)} \pi_B(l_1, l_2) = k_1 q_1 s_1 + k_2 q_2 s_2 - p(\Delta_1 + \Delta_2)$$

$$k_i l_i \leq s_i \leq \min\{b_i, d_i + \theta_i d_0 + \Delta_i\} \quad \text{for } i = 1, 2$$

$$\theta_1 + \theta_2 = 1$$

$$k_i \in \{0, 1\} \quad \text{for } i = 1, 2$$

$$0 \leq \Delta_i, \theta_i \quad \text{for } i = 1, 2$$

$\Delta_i$: Purchased units by the broker from provider $i$, in excess of demand to trigger a commission increase.

Providers’ Best Response Problem:

$$\max_{(l_i, q_i)} \pi_i(l_{3-i}, q_{3-i}) = (p - k_i q_i) s_i \quad \text{for } i = 1, 2$$

$$0 \leq l_i \leq b_i$$

$$0 \leq q_i \leq p$$

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Requiring Minimum Sales Volume for a Commission Increase
Theorem 2

In a competitive market, there exists a pure-strategy Nash equilibrium such that:

\[ q_i^* = \left( \frac{m_1 + m_2 - d}{m_i} \right) p \]

The equilibrium results in

\[ s_i^* = l_i^* = m_i \]
Magnitude of commission margins

In equilibrium, the primary provider pays smaller commission margins.
### Competitive market

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<th>$\frac{\delta \pi_i}{\delta v}$</th>
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<tbody>
<tr>
<td>$b_1 &lt; d_0 + d_1$</td>
<td>$\pi_1 = p \max[d - b_2, d_1]$</td>
<td>$&gt; 0$</td>
<td>$\leq 0$</td>
</tr>
<tr>
<td></td>
<td>$\pi_2 = p(d - b_1)$</td>
<td>$&gt; 0$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>$\pi_B = p(b_1 - \max[d - b_2, d_1])$</td>
<td>$&lt; 0$</td>
<td>$\geq 0$</td>
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<td>$&lt; 0$</td>
</tr>
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<td></td>
<td>$\pi_B = p(d_0 + d_1 - \max[d - b_2, d_1])$</td>
<td>$\in \mathbb{R}$</td>
<td>$&gt; 0$</td>
</tr>
</tbody>
</table>
Figure 2: Revenue when demand is not loyal

Figure: Flexible margins with thresholds
Figure 3: Revenues when demand is large and loyal

Figure: Flexible margins with or without thresholds
## Effect of the market demand and broker power

<table>
<thead>
<tr>
<th></th>
<th>Without thresholds</th>
<th>With thresholds</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>$\delta d$</td>
<td>$\delta d$</td>
</tr>
<tr>
<td>$v$</td>
<td>$\delta v$</td>
<td>$\delta v$</td>
</tr>
<tr>
<td>$\delta \pi_1$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
</tr>
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</tr>
<tr>
<td>$\delta \pi_B$</td>
<td>$\in \mathbb{R}$</td>
<td>$\in \mathbb{R}$</td>
</tr>
<tr>
<td>$\delta \pi_B$</td>
<td>$\geq 0$</td>
<td>$\geq 0$</td>
</tr>
</tbody>
</table>
Who loses and who wins in a competitive market

- The primary provider’s revenue remains “fixed”.
- The secondary broker “loses”.
- The broker “wins”.

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Strategic effects in a competitive market
Strategic effects in a competitive market

Without thresholds

\[ d_1 \quad d_0 \quad d_2 \]

\[ b_1 \quad b_2 \]

\[ p \]

\[ \pi_1 \quad \pi_B \quad \pi_2 \]
Strategic effects in a competitive market

Without thresholds

With thresholds
Paradox!

- Providers are not winning by introduction of thresholds.

- Yet, there is a big push by providers to introduce the thresholds.

- Why?!
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4 Conclusions and Further Research
Who loses and who wins in a competitive market \((q_1 > q_2)\)

- At least one of the providers “wins”.

<table>
<thead>
<tr>
<th>Condition</th>
<th>(b_2 &lt; d_0 + d_2)</th>
<th>(d_0 + d_2 &lt; b_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_1 &lt; d_0 + d_1)</td>
<td>(\hat{\pi}_1 \geq \hat{\pi}_2)</td>
<td>Fixed, Win</td>
</tr>
<tr>
<td></td>
<td>(\hat{\pi}_1 &lt; \hat{\pi}_2)</td>
<td>Loss, Win</td>
</tr>
<tr>
<td>(b_1 &gt; d_0 + d_1)</td>
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<tr>
<td></td>
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</tbody>
</table>

- The broker “loses”.

\[\text{Win unless } \hat{\pi}_1 = \hat{\pi}_2\]

\[\hat{s}_i = \max[m_i, \frac{p}{p-q_i}(d - m_j)]\]

\[\hat{\pi}_i = q_i\hat{s}_i - p(\hat{s}_i - m_i)\]
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The provider with the higher total commission fee, which in addition to commission margin depends on the available capacity and the loyal market too, gets prioritized.

The broker loses and the secondary provider wins. The primary one maybe wins or loses.

There is an incentive to introduce thresholds.

There are cases which discarding is inevitable.
Conclusions when margins are flexible

- There will be a pure equilibrium rather than a randomized one.
- Broker gains at expense of the secondary provider.
- Flexible margins with thresholds is the only stable equilibrium and the providers’ gains in fixed margins scenario are mirage.
Research opportunities

- Considering cost of production and distribution
- Considering other types of contracts
- Considering different prices
- Stochastic sale modelling
- Providers’ direct sale
- Providers’ asymmetrical strategies
THANK YOU!