Throughout, let \(X\) denote a complete CAT(0) space.

**Definition 0.1.** Following [1, Definition 5.13], we shall say that \(X\) has the property (N) if, given a geodesic \(\gamma \subset X\) and \(x, y \in X\), we have that \(P_\gamma(m)\) lies on the geodesic \([P_\gamma(x), P_\gamma(y)]\), for any \(m \in [x, y]\).

We remark that it is not known whether all complete CAT(0) spaces have the property (N), see [1].

**Lemma 0.2.** The property (N) is equivalent to the following property. Whenever a geodesic \(\gamma \subset X\) and \(x, y \in X\) are such that \(P_\gamma(x) = P_\gamma(y)\), then \(P_\gamma(z) = P_\gamma(x) = P_\gamma(y)\) for any \(z \in [x, y]\).

**Proof.** Easy. \(\square\)

**Definition 0.3.** Let \(\gamma \subset X\) be a geodesic and \(u \in \gamma\). The set \(\{x \in X : P_\gamma(x) = u\}\) is called a hyperplane.

**Definition 0.4.** Let \(\gamma \subset X\) be a geodesic and \(u_0, u_1 \in \gamma\). Suppose \(\rho \subset \gamma\) is the geodesic ray issuing from \(u_0\) and containing \(u_1\). The set \(\{x \in X : P_\gamma(x) \in \rho\}\) is called a halfspace.

The preceding definition is consistent with that one of the affine hyperplane/halfspace in Hilbert spaces.

**Proposition 0.5.** Suppose \(X\) has the property (N). Then all hyperplanes and halfspaces are convex.

**Proof.** Easy. \(\square\)

**Example 0.6.** Let \(X = \mathbb{R}^2 \setminus (0, \infty)^2\). The geodesic connecting \(x\) and \(y\) is the line segment (if it belongs to \(X\), or the concatenation of line segments \([x, 0]\) and \([y, 0]\).

Hence, \(X\) is a complete CAT(0) space with the property (N). If \(\gamma\) is the geodesic forming the boundary of \(X\) and \(u = (0, 0)\), then the set \((-\infty, 0] \times [0, \infty), [0, \infty) \times (-\infty, 0]\) are halfspaces.

**References**