often by publishing worksheets off the Sage public server. Sage will see significant action as I finish an integral calculus course this term with infinite series and Taylor polynomials. Preparing an upcoming presentation will give me an excuse to learn more about Sage’s graph theory routines.

Sage is big, and there is much to explore and to use in your professional activities as a mathematician. It is an impressive concentration and unification of mathematical knowledge. The reliance on mature open-source packages and open standards provides a measure of confidence and future-proofing. There are a few rough edges as the project matures, but this also provides the opportunity to get involved and influence development. But see for yourself by experimenting at the public server (sagemb.org) along with the 6,000 others who have accounts there, or simply install your own copy on your favorite hardware. Either way, it’s free.

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REFERENCES


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Tim Gowers was a 1998 Fields medalist for his marvellous resolution of long-standing problems in Banach space theory—such as whether it is possible for a Banach space to have no isomorphic hyperplane (it is)—and in combinatorics; and while he continues such work, in exemplary fashion he has also found time for various more didactic and expository projects such as Mathematics: A Very Short Introduction (2002) and the book under review, activity with various media, and much else. Both associate editors, June Barrow-Green (Deputy Director for the Centre and Research Fellow in History of Mathematics at the Open University) and the combinatorist Imre Leader (Professor at Trinity College, Cambridge), have distinguished records.

This work, which I shall refer to below, as Gowers does, as “The Companion,” is a fine validation of the well-known proposition that if you want a job done right you should ask a busy person to do it. In this case many very busy people have performed an invaluable job very, very well. This handsome, hefty, and attractively priced volume received Honorable Mention for the 2008 PROSE Award for Professional and Scholarly Excellence for Single Volume Reference/Science, Association of American Publishers. In his excellent preface Gowers describes the painstaking six-year process which led to this work and writes that “the central focus of this book is modern, pure mathematics,” both highlighted terms being lucidly discussed. Since this review is appearing in SIAM Review I should emphasize that a great deal of less pure mathematics is captured. He also points out that a “companion is not an encyclopaedia” and that
"The Princeton Companion to Mathematics could be said to be about everything that Russell’s definition of pure Mathematics leaves out."

Let me complete my review prematurely. Every research mathematician, every university student of mathematics, and every serious amateur of mathematical science should own a least one copy of The Companion. Indeed, the sheer weight of the volume suggests that it is advisable to own two: one for work and one at home. You may want to get a copy of The Companion for a friend. I bought a copy as an 85th birthday present for my mathematician father.

Reviews of The Companion, both professional and on Amazon.com (which also has a good selection of superlative comments extracted from professional reviews), have been generally laudatory, as indeed they should be. Princeton University Press also maintains a website at http://press.princeton.edu/TOCs/c8350.html (whose scope and intention Gowers describes in his preface) from which the careful potential buyer can make a fully informed decision to purchase. Additionally, Princeton University Press provides the full table of contents, the preface, the list of contributors (a most impressive collection which includes mathematical household names such as Atiyah, Connes, Daubechies, Lax, and Tao, as well as many very distinguished authors most of whose names are probably not familiar to any given reader), and very representative sample articles consisting of I.2 “The Language and Grammar of Mathematics,” II.2 “Geometry,” IV.5 “Arithmetic Geometry,” IV.21 “Numerical Analysis,” V.10 “Fermat’s Last Theorem,” VI.61 “Jules Henri Poincaré (1854–1912),” VII.2 “Mathematical Biology,” and VIII.6 “Advice to a Young Mathematician” (by Sir Michael Atiyah, Bela Bollobas, and others). From this list the reader of this review can already probably glean the structure of the book, which consists of eight parts.


Part II, “The Origins of Modern Mathematics,” has seven entries commencing with “From Numbers to Number Systems and Geometry” and culminating with “The Crisis in the Foundations of Mathematics.”


Part V, “Theorems and Problems,” has 36 alphabetic entries of between one and three pages. It starts as it must with “The ABC Conjecture,” and touches upon “The Banach–Tarski Paradox,” “Carleson’s Theorem,” “The Classification of Finite Simple Fermat’s Last Theorem,” “The Four-Color Theorem,” “Gödel’s Theorem,” “The Insolvability of the Halting Problem,” “Mostow’s Strong Rigidity Theorem,” “The

In Part VI, “Mathematicians” are arranged chronologically from Pythagoras (ca. 569 B.C.E.–ca. 494 B.C.E.) and Euclid (ca. 325 B.C.E.–ca. 265 B.C.E.) through Abu Ja’far Muhammad ibn Musa al-Khwarizmi (800–847), Leonardo of Pisa (known as Fibonacci) (ca. 1170–ca. 1250), François Viète (1540–1603), Pierre Fermat (1607–1665), the Bernoullis (fl. 18th century), Leonhard Euler (1707–1783), Jean-Baptiste Joseph Fourier (1768–1830), Carl Friedrich Gauss (1777–1855), Nicolai Ivanovich Lobachevskii (1792–1856), William Rowan Hamilton (1805–1865), Eduard Kummer (1810–1893), James Joseph Sylvester (1814–1897), William Burnside (1852–1927), Jacques Hadamard (1865–1963), Emmy Noether (1882–1935), Norbert Wiener (1894–1964), William Vallance Douglas Hodge (1903–1975), Abraham Robinson (1918–1974), and, finally, as the 96th entry and the only living member of the list, Nicolas Bourbaki (1935–). If your mathematical hero is missing above, that is likely to be because of my selection, not the editors’ oversight. Indeed, when I proofread this review I wondered why my own favorite G.H. Hardy was not listed above, but in fact he was indeed included.


Part VIII, “Final Perspectives,” comprises seven essays, each between about five and ten pages in length, entitled “The Art of Problem Solving,” “Why Mathematics? You Might Ask,” “The Ubiquity of Mathematics,” “Numeracy,” “Mathematics: An Experimental Science,” “Advice to a Young Mathematician” (perhaps in homage to Peter Medawar’s wonderful 1979 Advice to a Young Scientist), and “A Chronology of Mathematical Events.”

The sheer scale and scope of the book, which finishes with a very good index, should now be fully apparent. In my 2006 featured review in SIAM Review of The Oxford Users’ Guide to Mathematics (SIAM Rev., 48 (2006), pp. 585–594) I wrote generally of the issues involved with such projects and, despite great sympathy, I found much to be critical of with regards to the roles of both its editors and its publisher. Indeed, I wrote:

A more thorough review and production process would surely have adequately addressed this last set of issues. I can no better make this point than to quote Simkin and Fiske quoting [in Science] the late Stephen J. Gould in a review of Simon Winchester’s Krakatoa. (... These reviews do make me question the reliability of The Professor and the Madman.)

In his review of Winchester’s previous book, The Map That Changed the World, Stephen Jay Gould wrote: “I don’t mean to sound like an academic sourpuss, but I just don’t understand the priorities of publishers who spare no expense to produce an elegantly illustrated and beautifully designed book and then permit the text to waver in simple, straight–out factual errors, all easily corrected for the minimal cost of one scrutiny of the galleys by a reader with professional expertise....

With Krakatoa, the publisher clearly spared considerable expense, and this new book also wallows in errors. Perhaps, given our popular culture’s appetite for sensationalized disasters, a modern publisher would rather not see all those pesky details corrected.

Even an academic sourpuss should be pleased with the attention to detail of The Companion’s publishers, editors, and authors and with many judicious decisions—
about the level of exposition, level of detail, what to include and what to omit, and much more—which have led to a well-integrated and highly readable volume. Gowers writes:

[The editorial process has been a very active one: we have not just commissioned the articles and accepted whatever we have been sent. Some drafts have had to be completely discarded and new articles written in the light of editorial comments. Others have needed substantial changes, which have sometimes been made by the authors and sometimes by the editors. A few articles were accepted with only trivial changes, but these were a very small minority.]

I described in my 2006 review how hard it is to produce such a volume—let alone to do it so splendidly—and how easy it is to find fault in any project with such audacious goals. This I know full well from my own more prosaic efforts as a co-author of The Collins-Smithsonian Dictionary of Mathematics. Thus, in an attempt to limit bias, I left a copy in my office for several months and sampled it with students or colleagues who dropped in for a chat or with a query. I found little missing. Indeed, the only item I did not find during this process but thought I should have found was “Turing test,” and that term is perhaps not fairly within the compass of modern pure mathematics. I finish by quoting again from Gowers’ own Preface.

6 Who Is The Companion Aimed At?
The original plan for The Companion was that all of it should be accessible to anybody with a good background in high school mathematics (including calculus). However, it soon became apparent that this was an unrealistic aim: there are branches of mathematics that are so much easier to understand when one knows at least some university-level mathematics that it does not make good sense to attempt to explain them at a lower level. On the other hand, there are other parts of the subject that decidedly can be explained to readers without this extra experience. So in the end we abandoned the idea that the book should have a uniform level of difficulty.

Accessibility has, however, remained one of our highest priorities, and throughout the book we have tried to discuss mathematical ideas at the lowest level that is practical. In particular, the editors have tried very hard not to allow any material into the book that they do not themselves understand, which has turned out to be a very serious constraint [my emphasis]. Some readers will find some articles too hard and other readers will find other articles too easy, but we hope that all readers from advanced high school level onwards will find that they enjoy a substantial proportion of the book.

What can readers of different levels hope to get out of The Companion? If you have embarked on a university level mathematics course, you may find that you are presented with a great deal of difficult and unfamiliar material without having much idea why it is important and where it is all going. Then you can use The Companion to provide yourself with some perspective on the subject. (For example, many more people know what a ring is than can give a good reason for caring about rings. But there are very good reasons, which you can read about in RINGS, IDEALS, AND MODULES [III.81] and ALGEBRAIC NUMBERS [IV.1].)

If you are coming to the end of the course, you may be interested in doing research in mathematics. But undergraduate courses typically give you very little idea of what research is actually like. So how do you decide which areas of mathematics truly interest you at the research level? It is not easy, but the decision can make the difference between becoming disillusioned and ultimately not getting a Ph.D., and going on to a successful career in mathematics. This book, especially Part IV, tells
you what mathematicians of many different kinds are thinking about at the research
level, and may help you to make a more informed decision.

If you are already an established research mathematician, then your main use for
this book will probably be to understand better what your colleagues are up to. Most
non-mathematicians are very surprised to learn how extraordinarily specialized math-
ematics has become. Nowadays it is not uncommon for a very good mathematician to
be completely unable to understand the papers of another mathematician, even from
an area that appears to be quite close. This is not a healthy state of affairs: anything
that can be done to improve the level of communication among mathematicians is a
good idea. The editors of this book have learned a huge amount from reading the
articles carefully, and we hope that many others will avail themselves of the same
opportunity.

Judging by the sales numbers shown on Amazon, however they are actually com-
puted, a great many copies are already with readers, if perhaps not all yet read.
Everyone involved with this project deserves our deep gratitude.

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Simulation and Inference for Stochas-
tic Differential Equations. By Stefano M.
xvii+286 pp., hardcover. ISBN 978-0-387-
75838-1.

I jumped at the chance to review this book.
It deals with two themes that deserve a
much higher profile in applied and com-
putational mathematics: uncertainty and
inference. You don’t need to delve into
stochastic models in order to appreciate
their importance. A typical deterministic
model will involve initial data and physical
coefficients that are either

(a) completely unknown, or

(b) known only up to some level of error.

Case (a) would arise, for example, where
one or more rate constants in a chemical
reaction system could not be measured. In
this context, the unknown parameters could
be fitted to time series data relating to the
observed concentrations of various species.
A standard approach in computational and
applied mathematics is to treat this as an
optimization problem and seek the parameter
values that best fit the data, for example,
in a least squares sense subject to some
realistic constraints. However, the resulting
"point estimate" would be frowned upon
by many experts in statistical inference [2],
who would argue, quite reasonably, that re-
turning just a single number (or even a sin-
gle number plus some sort of local sensitivity
estimate) is an inadequate summary, with
the language and tools of probability theory
providing a more appropriate setting.

Case (b) could of course arise after ob-
served data has been used to deal with
case (a). It may also arise when coefficients
can be observed directly, but the measure-
ments are subject to experimental errors.
In either circumstance, it seems illogical to
focus all our energies on theoretical or nu-
erical analysis of a single "best guess" of
the underlying problem specification. In-
stead, we should deal with questions such
as: Given a quantitative representation of
the uncertainty in the model, can we find
a quantitative representation of the uncen-
tainty in the output? Of course, analyzing
or computing the solution for a fixed in-
stance of the model will be an important
subproblem. But the bigger picture, which
sits at the intersection between statistics,
probability, computer science, and applied
mathematics, raises many new challenges
and, in my opinion, deserves much greater
attention.

I hope I have made clear my view that
uncertainty and inference have a signifi-