Referee’s Report of “Maximality of sums...” by J. Borwein

This paper contains a fundamental result in convex analysis and definitely deserves publication in the Proceedings of the AMS. Unfortunately, it has been written in a very informal style, much like one expert talking to another in front of a blackboard, and will require considerable rewriting before it can be accepted for publication. A number of suggested revisions are given below, designed to make the paper accessible to more readers than the handful who have been working on the problem.

In the Introduction and Preliminaries, it should be noted that $X$ is a Banach space. Further, $D(B)$ should be defined. Towards the end of the third paragraph, the word “covered” should be replaced by “defined”.

p.2, line 2: should read: Fitzpatrick’s function is defined on $X \times X^*$ by

p.2, line 8: should read: Penot’s function on $X \times X^*$ is given ...

p.2, line 11: ... (and the conjugate $P_T$ defined on $(X \times X^*)^*$ ... The author should include some words explaining why $X \times X^*$ can be considered as a subspace of $(X \times X^*)^*$

p.2, line 13: “lower–semicontinuous hull” has not been defined

p.2, line 15: define “representative”, perhaps by writing ... represents a monotone operator $T$ (or is a representative for $T$) if ...

p.2, Prop. 2 (iii): If $T$ is maximal and $H_T$ represents $T$, then ...

p.2, in Proof: (ii.) is a direct consequence of $\mathcal{T} = (CT)^{**}$ and the fact that ...

p.2, last para: Since this appears at the bottom of the page, the reader is left wondering why “The situation is however ameliorated ...” It might be better to imply that there is more to follow, by starting with something like “We can show that the situation ...”

p.3, line 5: ... in its own right; it is a ...

p.3, Theorem 3: “attained when finite” is undefined. (The same phrase is used in the first paragraph of the proof.)

p.3, Proof of Theorem 3: first para: “direct Langrangian calculation” is undefined.

p.3, middle of page: definition of “almost maximal”: The operator $S$ is not defined.

p.3, Corollary 4: Suppose $T$ is monotone and almost maximal ... closed convex function $H$ on $X \times X^*$ represents ...

p.3, Proof of Cor 4 second line: Suppose we show that $H^*$ (restricted to $X \times X^*$) is also ...

p.3, last line: maximal whenever it is almost maximal. (“as soon as” suggests that it is in a race!)

p.4, line 6: “close–to–maximal” is undefined.

p.4, para preceding Sec. 3, 2nd line: change “already” to “even”. 3rd line: change “proven” to “proved”.

p.4, preceding Prop 8: The awkward ordering in the definition of algebraic closure forced the reviewer to read it a couple of times. Why not \( \{ d; \exists 0 < t_n < 1, t_n \to 1 \text{ such that } t_n d + (1 - t_n)x \in C \} \)?

p.4, line (-6): “implies that the normal cones satisfy \( N_U + N_V \ldots \)

p.5, 2nd line of Proof of Theorem 9: \ldots nonempty interior; see [2, 9, 10]. Moreover,

p.5, second line of second para of Proof of Theorem 9: This sentence should be re-written; it was incomprehensible to the reviewer.

p.5, fifth line of Proof of Theorem 10: \( A_F, B_F \) do not appear to have been defined.

p.6, first line of Remark 12: \ldots and convex in a Banach space

p.6, following Remark 13: Usual style is to say “We conjecture \ldots” What does “in the extreme cases” mean?

p.6, The Proof of Theorem 14 should not be in italics.

p.7, first line: \ldots still ensures that \ldots represents \( T + \partial f \), as it

p.7, sentence prior to Corollary 16: Unneeded comma after \( y^* \in T(y) \).

p.7, statement of Corollary 16: \ldots closed and convex; then \( T \ldots \)