Integrated Mine Planning IMP

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This Presentation

- Project background
- Summary of the work done in 2007
- Future directions
Background & Participants

- The IMP project it is part of CDSC’s Industrial Control and Optimisation Programme.

- It started in 2003 as a consequence of a scope revision of the predecessor project Mine to Market Optimization (MMO).

- The IMP project focuses on the optimization of the life-of-mine net present value by scheduling activities while taking into account factors such as forward price estimates, risk models, and ore body uncertainty.

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Mine Process and IMP Objectives

**Information**
- Metal Price (est.)
- Ore Body Content and location (est.)
- Plant capital & operating costs
- Operating Constraints

**Decision Variables**
- Process plant & fleet size
- Reserve Size
- Cutoff Grade(s) (vs t)
- Extraction schedule (vs t)

**Outputs**
- Mining ‘phase’ design
- Cash flow/NPV

\[
NPV = \sum_{i=1}^{N} \frac{C_i}{(1 + d)^i}
\]
Mine Block Model

Block model (2D abstraction):

Scheduling Problem:

*When do we extract the each block?*
Procedure: Step 1—Ultimate Pit

Find *ultimate* pit (opening left after the mining is completed)
Procedure: Step 2—Clumps

Group Blocks together into *aggregates* and schedule aggregates

Blocks spatially connected and with similar properties are grouped. The *aggregates* obey slope constraints.
Procedure: Step 3—Phase design

Reform into *phases* and *panels*, which are more manageable planning units, then these are scheduled.

Panel = Bench $\cap$ Phase

Panels from different phases can be mined at the same time.
Scheduling Problem

The scheduling problem can be posed as a constrained optimization problem:

$$\text{Max } J(\text{NPV})$$

- **Decision variables**: $z(i,t)$: 1 if block $i$ is extracted in the periods 1 to $t$, 0 otherwise.

- **Constraints**:
  - wall slope,
  - equipment capacity
  - dynamics (extraction)
  - precedence

- **Disturbances**:
  - price and cost fluctuations over the life of the mine
Work done in 2007

- Alternative definition of scheduling problem as an stochastic optimal control problem.

- Consideration of closed-loop solutions and value of information.

- Study of receding horizon optimization as a practical way to solve the problem.

- Modelling of uncertainty commodity price values in terms of scenarios so as to simplify solutions.
Stochastic Optimal Control

The 3 ingredients:

- Underlying stochastic dynamic system
- Constraints
- Objective function (to optimise)
Stochastic dynamic system

\[ x_{k+1} = f_k(x_k, u_k, w_k) \]

\[ x_k \in X_k, \quad u_k \in U_k \]

Here \( k \) can represent time and,

- \( x_k \) – state: information about the process.
- \( u_k \) – decision variable, which depends on the information \( x_k \).
- \( w_k \) – uncertainty—stochastic process.
- \( f_k \) – governing transitions.
- \( X_k, U_k \) constraint sets.
Objective function

\[ g_T(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k) \]

- The objective function represents a desired effect or outcome; and therefore we seek to maximise it.

- The objective function is additive: the cost accumulates over time \( k \).

- Because of the uncertainty introduced by \( w_k \), it only makes sense to optimise its expected value:

\[ J = E \left\{ g_T(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k) \right\} \]
Solution Approaches

The optimisation is performed over the decision variables:

\[ u_0, u_1, \ldots, u_{N-1} \]

There are two alternatives to solve this kind of problems:

- Open loop
- Closed loop
Open-Loop Approach

The value of all the decision variables are chosen at once at the initial stage $k=0$, without accounting for the fact that at the different stages, there will be further information available for the decision:

$$u^{opt} = \left\{ u_0^{opt}, u_1^{opt}, \ldots, u_{N-1}^{opt} \right\}$$

$$u^{opt} = \arg \min_{u_1, \ldots, u_{N-1}} E \left\{ g_T(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k) \right\}$$

$$x_{k+1} = f_k(x_k, u_k, w_k) \quad x_k \in X_k, \quad u_k \in U_k$$
Closed-Loop Approach

The optimisation is done over \textit{policies} (functions) rather than values:

\[
\pi_{opt} = \left\{ \pi_{0opt}(\cdot), \pi_{1opt}(\cdot), \ldots, \pi_{N-1opt}(\cdot) \right\}
\]

\[
\pi_{opt} = \arg \min_{\pi_1, \ldots, \pi_{N-1}} E \left\{ g_T(x_N) + \sum_{k=0}^{N-1} g_k(x_k, \pi_k(x_k), w_k) \right\}
\]

\[
x_{k+1} = f_k(x_k, u_k, w_k) \quad x_k \in X_k, \quad u_k \in U_k
\]

Once we have the policies, when new information is available, the decision is made:

\[
u_{kopt} = \pi_{kopt}(x_k)
\]
What is the difference?

- In deterministic problems, both approaches lead to the same solution—no difference.

- In uncertain problems; however, there can be a significant difference, which is attributed to the *value of information*.

- CL-solutions allow being flexible and changing the decision depending on current information.
Mining: dynamics

A dynamic model

\[ x_{jk}(t+1) = x_{jk}(t) + b u_{jk}(t) \]

- \(x_{jk}(t)\) mine depth at location \(j,k\) at time \(t\).
- \(u_{jk}(t)\) action to mine or not to mine.
- \(b\) reflects the effect of one unit of mining action.
Mining: Objective function

\[ J = \sum_{t,j,k}^{T,J,K} d_t c_t V_{jk} [x_{jk}(t)] u_{jk}(t) \]

- \(c_t\) price of ore at time \(t\)—uncertainty.
- \(d_t\) discount coefficient to yield the NPV.
- \(V_{jk}\) reflects cost of mining, and this is multiplied by the mining action to since the ore is only liberated when it is mined.
A toy example

Objective: \[ J = E \left\{ \sum_{t=1}^{T} c_t [x_0(t) - x_0(t - 1)] \right\} \]

System:
\[ x_k(t+1) = x_k(t) + b u_k(t) \]

Constraints:
\[ x_k(0) = 0; \quad \text{for all } k \in \{0, \ldots, K\} \]
\[ x_k(t) \geq x_k(t - 1); \quad \text{for all } k \in \{0, \ldots, K\}, \ t \in \{1, \ldots, T\} \]
\[ |x_{k-1}(t) - x_k(t)| \leq 1; \quad \text{for all } k \in \{1, \ldots, K\}, \ t \in \{1, \ldots, T\} \]
\[ \frac{x_0(t) - x_0(t - 1) + x_K(t) - x_K(t - 1)}{2} + \sum_{k=1}^{K-1} [x_k(t) - x_k(t - 1)] \leq \frac{C}{2}; \quad \text{for all } t \in \{1, \ldots, T\} \]
Practical solutions

- Optimising over policies requires the application of *Dynamic Programming*, and this can be a daunting task—curse of dimensionality.

  (1000 decisions at each step over 15 steps gives $15^{1000}$ cases, and the known number of atoms in the universe is $10^{80}$.)

- A practical approach consists of selecting representative scenarios for the uncertain variables and solve and open-loop problem whenever new information is available: *a receding horizon approach.*
Receding Horizon

\[ U_{OPT} = \{ u_0^{OPT}, \ldots, u_{N-1}^{OPT} \} \]

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Time
Uncertainty in Ore Price

Cu price and trend

De-trended

Stochastic models can be used to describe the residuals obtained by de-trending the price data.
Uncertainty in Ore Price

- The literature of statistics of financial markets provides different real-valued stochastic models
  - Geometric Brownian Motion
  - Random walk with mean reversion
  - Autoregressive conditional Heteroskedasticity (ARCH)

- These models are appropriate to generate sample realizations for simulation, but seem too complex for planning and optimization.

- A more viable approach consists of selecting representative scenarios for close-loop planning—via approximations leading to discrete-valued stochastic processes.
Scenario Generation

- Proposed methods work by approximating continuous high-dimensional distributions by discrete ones.

- A new method has been developed for the mine planning: Hinged stochastic interpolation.
Scenario Generation

Hinged stochastic interpolation is based on a time and space quantization.

$5^3 = 125$ scenarios.
1- Chose hinge points (where the process will pass through) to cover the range of possible values—this is a key issue.
2- Chose the values at the remaining scenarios at the stages with no hinge points via optimization.
3- Assign a probability to each scenario.
Example

HIS Cu Price from 1994 to 2002

- green: true price
- black: closest scenario in MSE
Scenarios

- Generating scenarios is much faster than tossing the coin (Monte Carlo simulation).
- The key idea behind it is a smart location of the hinge points.
- Scenarios are not to be used to forecast the commodity price, but to optimise policies for the decisions.
Back to the Toy example

We consider only 4 scenarios:

\[ s \in \{1, 2, 3, 4\} \]

\[ P\{c_2 = v_2\} = \alpha \]
\[ P\{c_2 = v_3\} = 1 - \alpha \]
\[ P\{c_3 = v_4 | c_2 = v_2\} = \beta \]
\[ P\{c_3 = v_5 | c_2 = v_2\} = 1 - \beta \]
\[ P\{c_3 = v_6 | c_2 = v_3\} = \gamma \]
\[ P\{c_3 = v_7 | c_2 = v_3\} = 1 - \gamma \]
Two price cases

\[ v_1 = 0.5 \]
\[ v_2 = 0.1 \]
\[ v_3 = 10 \]
\[ v_4 = 0.01 \]
\[ v_5 = 0.25 \]
\[ v_6 = 1 \]
\[ v_7 = 25 \]

\[ \alpha = \frac{32}{33} \]
\[ \beta = \frac{17}{24} \]
\[ \gamma = \frac{17}{24} \]

\[ v_1 = 0.41 \]
\[ v_2 = 0.1 \]
\[ v_3 = 10.5 \]
\[ v_4 = 0.01 \]
\[ v_5 = 0.25 \]
\[ v_6 = 1 \]
\[ v_7 = 25 \]

\[ \alpha = \frac{32}{33} \]
\[ \beta = \frac{17}{24} \]
\[ \gamma = \frac{7}{12} \]
Case 1

There is a tendency for the price to decrease—it does not matter how much they decrease, lets dig!!!
Here, we can appreciate a difference in policy: while receding horizon continues digging, the closed loop solution prepares the ground to be able to extract more metal in anticipation for metal price increase.

\[ c_1 = v_1 = 0.41 > 0.40588 \approx E\{c_2\} \]
\[ E\{c_2\} \approx 0.40588 > 0.40118 \approx E\{c_3\} \]
\[ v_2 = 0.1 > 0.08 = E\{c_3|c_2 = v_2\} \]
\[ v_3 = 10.5 < 11 = E\{c_3|c_2 = v_3\} \]
Future work

- Optimal selection of hinge points.
- Viewing scenarios as “code-book design” as in vector quantization.
- Study condition under which closed-loop and RH give significantly different answers.
- Optimization based on set-membership description of uncertainty.