High Performance Mathematics and Maple

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Canada Research Chair in Collaborative Technology
[Background: Optimization, Analysis, Number Theory, Computational Math]

“I feel so strongly about the wrongness of reading a lecture that my language may seem immoderate .... The spoken word and the written word are quite different arts .... I feel that to collect an audience and then read one's material is like inviting a friend to go for a walk and asking him not to mind if you go alongside him in your car.”

Sir Lawrence Bragg

What would he say about .ppt?

Director of AARMS
East meets West: Collaboration goes National

Welcome to D-DRIVE whose mandate is to study and develop resources specific to (‘dis-located’) distributed research and interaction in the sciences with first client groups being the following communities:

- High Performance Computing
- Mathematical and Computational Science Research
- Science Outreach
  - Research
  - Education
  - Media

Atlantic Computational Excellence Network (ACEnet)
D-DRIVE  Jon Borwein  P. Borwein (SFU)  D. Bailey (Lawrence Berkeley)
R. Crandall (Reed and Apple) and many others

Staff  David Langstroth (Manager)  Scott Wilson (Systems)
       Peter Dobscanyi (HPC)

Students  Macklem (Parallel Optimization)  Wiersma (Analysis/NIST)
          Hamilton (Inequalities and Computer Algebra)  Ye (Quadrature)
          Paek (Federated search)  Oram (Haptics)

AIM: (Secure, Stable, Satisfying) Presence at a Distance
Content Provider: putting math and science on handhelds, laptops, web, in classrooms …
MRI’s First Product in Mid-nineties

PAVCA SED MATVRA

MathResourses Inc.

► Built on Harper Collins college dictionary - an IP adventure!
► Maple inside the MathResource
► Database now in Maple 9.5/10
► CONVERGENCE?
"It says it's sick of doing things like inventories and payrolls, and it wants to make some breakthroughs in astrophysics."
Outline of HPM Talk

A. Communication, Collaboration and Computation.

B3. Inverse Symbolic Computation.

C. Wish-list and Demos.

Challenges of MKM (Math Knowledge Management)
- integration of tools, inter-operability
- e.g., workable mathematical OCR
- intelligent-agents, automated use
- many IP/copyright and sociological issues
- metadata, standards and on

www.mkm-ig.org
Experiments in Mathematics

Jonathan M. Borwein
David H. Bailey
Roland Girgensohn
Produced with the assistance of Mason Macklem

The reader who wants to get an introduction to this exciting approach to doing mathematics can do no better than these books.
—Notices of the AMS

I do not think that I have had the good fortune to read two more entertaining and informative mathematics texts.
—Australian Mathematical Society Gazette

This Experiments in Mathematics CD contains the full text of both Mathematics by Experiment: Plausible Reasoning in the 21st Century and Experimentation in Mathematics: Computational Paths to Discovery in electronic, searchable form. The CD includes several "smart" enhancements, such as

- Hyperlinks for all cross references
- Hyperlinks for all Internet URLs
- Hyperlinks to bibliographic references
- Enhanced search function, which assists one with a search for a particular mathematical formula or expression.

These enhancements significantly improve the usability of these files and the reader's experience with the material.

ISBN 1-56881-293-3

Jonathan M. Borwein, David H. Bailey, Roland Girgensohn
Produced with the assistance of Mason Macklem

A K Peters, Ltd.
Experimental Methodology

1. Gaining insight and intuition
2. Discovering new relationships
3. Visualizing math principles
4. Testing and especially falsifying conjectures
5. Exploring a possible result to see if it merits formal proof
6. Suggesting approaches for formal proof
7. Computing replacing lengthy hand derivations
8. Confirming analytically derived results
Grand Challenges in Mathematics (CISE 2000)

are few and far between

• **Four Colour Theorem** (1976, 1997)
  • Kepler’s problem (Hales, 2004-12)

On an upcoming slide

• **Nonexistence of Projective Plane of Order 10**
  – $10^2 + 10 + 1$ lines and points on each other (n+1 fold)
    • 2000 Cray hrs in 1990
    • next similar case: 18 needs $10^{12}$ hours?
    • or a Quantum Computer

**Fermat’s Last Theorem** (Wiles 1993, 1994)
  – By contrast, any counterexample was too big to find (1985)

\[ x^N + y^N = z^N, \quad N > 2 \]

has only trivial integer solutions
The first series below was proven by Ramanujan. The next two were proven by Computer (Wilf-Zeilberger).

\[ \frac{32}{\pi^3} = \sum_{n=0}^{\infty} r_7(n) \left( 168n^3 + 76n^2 + 14n + 1 \right) \left( \frac{1}{4^3} \right)^n \]

Here, in terms of factorials and rising factorials:

\[ r_N(n) := \frac{(2n)^N}{4nN} = \left( \frac{(1/2)_n}{n!} \right)^N. \]

4th is only discovered/true

\[ r_N(n) \sim n \frac{1}{n^{N/2}} \]
Formal Proof theory (code validation) has received an unexpected boost: automated proofs may now exist of the Four Color Theorem and Prime Number Theorem

- **Kepler's** conjecture the densest way to stack spheres is in a pyramid
  - oldest problem in discrete geometry?
  - most interesting recent example of computer assisted proof
  - published in *Annals of Mathematics* with an “only 99% checked” disclaimer
  - Many varied reactions. *In Math, Computers Don't Lie. Or Do They?* (NYT, 6/4/04)

- **Famous earlier examples**: Four Color Theorem and Non-existence of a Projective Plane of Order 10.
  - the three raise quite distinct questions - both real and specious
  - as does status of classification of Finite Simple Groups

**Formal Proof theory** (code validation) has received an unexpected boost: automated proofs *may* now exist of the Four Color Theorem and Prime Number Theorem

- **COQ**: *When is a proof a proof?* Economist, April 2005
Fermat's Last Theorem for 500 miles.
Components include

- **AccessGrid**
- **UCLP** for
  - haptics
  - learning objects
  - visualization
- **Grid Computing**
Six degrees of net separation ...
CoLab at SFU (2001)

Me and my Avatar (2003)

All features are active
Coast to Coast Seminar Series (C2C)

Lead partners:
- Dalhousie D-Drive – Halifax, Nova Scotia
- IRMACS – Burnaby, British Columbia

Other Participants so far:
- University of British Columbia, University of Alberta, University of Alberta University of Saskatchewan, Lethbridge University, Acadia University, St Francis Xavier University, University of Western Michigan, MathResources Inc, University of North Carolina

Tuesdays 3:30 – 4:30 pm Atlantic Time

http://projects.cs.dal.ca/ddrive/
The Experience

Fully Interactive multi-way audio and video

audio is harder (given good bandwidth)

The closest thing to being in the same room

Shared Desktop for viewing presentations or sharing software
The 2,500 sq-metre IRMACS research centre

SFU building is also a 190cpu G5 Grid

At the official April 2005 opening, I gave one of the four presentations from D-DRIVE

Trans-Canada ‘C2C’ Seminar
Tuesdays PST 11.30 MST 12.30 AST
3.30 and even 7.30 GMT
[Sept 28 - PBB on RH]
Jonathan Borwein, Dalhousie University
Mathematical Visualization

Uwe Glaesser, Simon Fraser University
Semantic Blueprints of Discrete Dynamic Systems

Peter Borwein, IRMACS
The Riemann Hypothesis

‘No one explains chalk’

Arvind Gupta, MITACS
The Protein Folding Problem

Jonathan Schaeffer, University of Edmonton
Solving Checkers

Przemyslaw Prusinkiewicz, University of Calgary
Computational Biology of Plants

Karl Dilcher, Dalhousie University
Fermat Numbers, Wieferich and Wilson Primes
The Technology

High Bandwidth Connections (CA*net)
+ PC Workstations
+ Audio/Video Equipment
+ Open Source Software

www.accessgrid.org
Institutional Requirements
(Scalable Investment)

Personal Nodes
(1-4 people)
Cost: Less than $10,000 (CA)

Small Group
Projected Environment
(2-10 people)
Cost: $25,000 - $100,000 (CA)

Meeting Room
Interactive Environment
(2-20 people)
Cost: $150,000 (CA)

Visualization Auditorium
Cost: $500,000+ (CA)

One Collaboration Experience
Haptics in the MLP

Haptic Devices extend the world of I/O into the tangible and tactile

To test latency issues

We link multiple devices so two or more users may interact at a distance (BC/NS Demo in April)

- in Museums and elsewhere
- Kinesiology, HCI

D-DRIVE Doug our haptic mascot

Sensible’s Phantom Omni
My remaining intention is to show a variety of mathematical uses of high performance computing and communicating as part of Experimental Inductive Mathematics.

Our web site:

www.experimentalmath.info

contains all links and references

"Elsewhere Kronecker said "In mathematics, I recognize true scientific value only in concrete mathematical truths, or to put it more pointedly, only in mathematical formulas." ... I would rather say "computations" than "formulas", but my view is essentially the same."

Harold Edwards, Essays in Constructive Mathematics, 2004
Very cool for the one person with control
"What I appreciate even more than its remarkable speed and accuracy are the words of understanding and compassion I get from it."
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B3. Inverse Symbolic Computation.
C. [Wish-list](#) and Demos.

The talk ends when I do
COXETER’S (1927) Kaleidoscope
The Perko Pair $10_{161}$ and $10_{162}$

are two adjacent 10-crossing knots (1900)

- first shown to be the same by Ken Perko in 1974
- and beautifully made dynamic in [KnotPlot](https://knotplot.com) (open source)
An unusual Mandelbrot parameterization

Various visual examples follow

- Indra’s pearls
- Roots of `1/-1’ polynomials
- Ramanujan’s fraction
- Sparsity and Pseudospectra

AK Peters, 2004
(CD in press)
Indra’s Pearls
A merging of 19th and 21st Centuries

DOUBLE CUSP GROUP

2002: http://klein.math.okstate.edu/IndrasPearls/
CINDERELLA

FOUR DEMOS combining inversion, reflection and dilation

1. Indraspearls
2. Apollonius
3. Hyperbolicity
4. Gasket

www.cinderella.de
Striking fractal patterns formed by plotting complex zeros for all polynomials in powers of $x$ with coefficients 1 and -1 to degree 18

Coloration is by sensitivity of polynomials to slight variation around the values of the zeros. The color scale represents a normalized sensitivity to the range of values; red is insensitive to violet which is strongly sensitive.

- All zeros are pictured (at 3600 dpi)
- Figure 1b is colored by their local density
- Figure 1d shows sensitivity relative to the $x^9$ term
- The white and orange striations are not understood

A wide variety of patterns and features become visible, leading researchers to totally unexpected mathematical results

"The idea that we could make biology mathematical, I think, perhaps is not working, but what is happening, strangely enough, is that maybe mathematics will become biological!"  
Pictures are more democratic but they come from formulae.
Roots in the most stable colouring
“Mathematics and the aesthetic
Modern approaches to an ancient affinity”
(CMS-Springer, 2006)

Why should I refuse a good dinner simply because I don't understand the digestive processes involved?

Oliver Heaviside (1850 - 1925)

- when criticized for his daring use of operators before they could be justified formally
Experimentation with DGEMV (matrix-vector multiply):

128x128=16,384 cases.

Experiment took 30+ hours to run.

Best performance = 338 Mflop/s with blocking=11 unrolling=11

Original performance = 232 Mflop/s
Numeric and Symbolic Computation

- Central to my work - with Dave Bailey - meshed with visualization, randomized checks, many web interfaces/databases (NIST)
- Massive (serial) Symbolic Computation - Automatic differentiation code
  - Integer Relation Methods
  - Inverse Symbolic Computation

Parallel derivative free optimization in Maple

Other useful tools: Parallel Maple
- Sloane’s online sequence database
- Salvy and Zimmerman’s generating function package ‘gfun’
- Automatic identity proving: Wilf-Zeilberger method for hypergeometric functions
Maple on SFU 192 cpu ‘bugaboo’ cluster
2002 - different node sets are in different colors
§A1.4. Maclaurin Series

\[ A_1(z) = A_1(0) \left(1 + \frac{1}{3}z^3 + \frac{14}{61}z^6 + \frac{147z^9}{91} + \cdots \right) + A_1'(0) \left(z + \frac{2}{4}z^4 + \frac{25}{71}z^7 + \frac{258z^{10}}{101} + \cdots \right) \]

Symbols used:
- \( \text{AiryAi} \)
- \( \text{cdots} \)

A&S Ref:
- 10.4.2 (with 10.4.4 and 10.4.5)

Encodings:
- \( \text{LaTeX} \)
- \( \text{MathML} \)

\[ \begin{align*}
\text{AiryAi}(z) &= \text{AiryAi}(0) \left(1 + \left(\frac{1}{3}\right)z^3 + \left(\frac{14}{61}\right)z^6 + \left(\frac{147}{91}\right)z^9 + \cdots \right) \\
&+ \text{AiryAi}'(0) \left(z + \left(\frac{2}{4}\right)z^4 + \left(\frac{25}{71}\right)z^7 + \left(\frac{258}{101}\right)z^{10} + \cdots \right) \\
\end{align*} \]
We have certain knowledge without proof. The easiest of 998 empirical PSLQ results linking physics/topology (LHS) to number theory (RHS). [JMB-Broadhurst]

```
\frac{24}{7\sqrt{7}} \int_{\pi/3}^{\pi/2} \log \left| \frac{\tan t + \sqrt{7}}{\tan t - \sqrt{7}} \right| \, dt = L_{-7}(2) \quad (\oplus)
```

where

\[ L_{-7}(s) = \sum_{n=0}^{\infty} \left[ \frac{1}{(7n+1)^s} + \frac{1}{(7n+2)^s} - \frac{1}{(7n+3)^s} + \frac{1}{(7n+4)^s} - \frac{1}{(7n+5)^s} - \frac{1}{(7n+6)^s} \right]. \]

“Identity” (\oplus) has been verified to 20,000 places. I have no idea of how to prove it.
Expected and unexpected scientific spinoffs

• **1986-1996.** Cray used quartic-Pi to check machines in factory
• **1986.** Complex FFT sped up by factor of two
• **2002.** Kanada used hex-pi (20hrs not 300hrs to check computation)
• **2005.** Virginia Tech (this integral pushed the limits)
• **1995-** Math Resources (another lecture)

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The integral was split at the nasty interior singularity. The sum was `easy'. All fast arithmetic & function evaluation ideas used.
"What it comes down to is our software is too hard and our hardware is too soft."
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C. Wish-list and Demos.

The talk ends when I do
Let \((x_n)\) be a vector of real numbers. An integer relation algorithm finds integers \((a_n)\) such that
\[
a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = 0
\]

- At the present time, the PSLQ algorithm of mathematician-sculptor Helaman Ferguson is the best-known integer relation algorithm.
- PSLQ was named one of ten “algorithms of the century” by *Computing in Science and Engineering*.
- High precision arithmetic software is required: at least \(d \times n\) digits, where \(d\) is the size (in digits) of the largest of the integers \(a_k\).

**An Immediate Use**

To see if \(a\) is algebraic of degree \(N\), consider \((1, a, a^2, \ldots, a^N)\)
Peter Borwein in front of Helaman Ferguson’s work

CMS Meeting
December 2003
SFU Harbour Centre

Ferguson uses high tech tools and micro engineering at NIST to build monumental math sculptures
PSLQ and Zeta

1. via PSLQ to 50,000 digits (250 terms)

\[ \zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \]

\[ \zeta(4) = \frac{\pi^4}{90}, \zeta(6) = \frac{\pi^6}{945}, \ldots \]

2005 Bailey, Bradley & JMB discovered and proved - in Maple - three equivalent binomial identities

\[ \zeta(x) = 3 \sum_{k=1}^{\infty} \frac{1}{\binom{2k}{k} (k^2 - x^2)} \prod_{n=1}^{k-1} \frac{4x^2 - n^2}{x^2 - n^2} \]

\[ = \sum_{k=0}^{\infty} \zeta(2k + 2) x^{2k} = \sum_{n=1}^{\infty} \frac{1}{n^2 - x^2} \]

\[ = \frac{1 - \pi x \cot(\pi x)}{2x^2} \]

2. reduced as hoped

\[ 3n^2 \sum_{k=n+1}^{2n} \frac{\prod_{m=n+1}^{k-1} (\frac{4n^2 - m^2}{n^2 - m^2})}{\binom{2k}{k} (k^2 - n^2)} = \frac{1}{\binom{2n}{n}} - \frac{1}{\binom{3n}{n}} \]

3. was easily computer proven (Wilf-Zeilberger) human proof (MAA)?
My brother made the observation that this log formula allows one to compute binary digits of log 2 without knowing the previous ones! (a BBP formula)

This reduced to which Maple, Mathematica, and humans can easily prove. A triumph for "reverse engineered mathematics" [algorithm design]

Bailey, Plouffe and he hunted for such a formula for Pi. Three months later the computer - doing bootstrapped PSLQ hunts - returned:

No such formula exists base-ten (provably)

Finalist for the $100K Edge of Computation Prize won by David Deutsch
IF THERE WERE COMPUTERS IN GALILEO'S TIME.
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Inverse Symbolic Computation

Inferring mathematical structure from numerical data

Mixes large table lookup, integer relation methods and intelligent preprocessing – needs *micro-parallelism*

- It faces the “curse of exponentiality”
- Implemented as Recognize in Mathematica and identify in Maple

Inverse Symbolic Calculator

Expressions that are not numeric like \( \ln(\pi \times \sqrt{2}) \) are evaluated in Maple in symbolic form first, followed by a floating point evaluation followed by a lookup.
Knuth’s Problem

Donald Knuth* asked for a closed form evaluation of:

\[ \sum_{k=1}^{\infty} \left\{ \frac{k^k}{k! e^k} - \frac{1}{\sqrt{2 \pi k}} \right\} = -0.084069508727655 \ldots \]

The answer is Gonnet’s Lambert’s W which solves \( W \exp(W) = x \).

"instrumentation"

Arguably we are done.

W's Riemann surface

The answer could be computed in 20 or 200 digits.

\( W \)'s Riemann surface is shown on next slide.

The solution in the Inverse Sym-
turns

\[ \approx \frac{2}{3} + \frac{\zeta(1/2)}{\sqrt{2\pi}}. \]

Which Maple 9.5 on a

\[ \text{in under 6 seconds} \]

* Arguably we are done.
ENTERING

\[ \text{evalf(Sum(k^k/k!/exp(k)-1/sqrt(2*Pi*k), k=1..infinity), 16)} \]

'Simple Lookup' fails; 'Smart Lookup' gives:

\[ \text{.08406950872765600} \]

\[ \text{= K} \]

\[ \text{Mixed constants with 5 operations} \]
\[ \text{5825971579390106 = zeta(1/2)/Si(2)/Si(Pi)} \]

\[ \text{Browse around .5825971579390106.} \]
Solid-state Physics: Ising Integrals

Analytical Ising theory of magnetic susceptibility involves multidimensional integrals such as

\[ D_3 := \frac{1}{3!} \int \frac{\tanh^2 \left( \frac{x-y}{2} \right) \tanh^2 \left( \frac{x-z}{2} \right) \tanh^2 \left( \frac{z-y}{2} \right)}{(\cosh x + \cosh y + \cosh z)^2} \, dx \, dy \, dz, \]

which integral is the “ferromagnetic constant.” Through hard work, one actually knows this entity, as:

\[ D_3 = 8 + \frac{4}{3} \pi^2 - 27 \, \text{Li}_3(2) \]
\[ = 0.0643073865806814763652607... \]

We also know \( D_4 \) in terms of Riemann-zeta values, but a complete theory requires all \( D_n \).

With Bailey and Crandall
We study the **first perturbation term** of the Ising $D_n$, as the $n$-dimensional integral

$$C_n := \frac{1}{n!} \int \frac{dx_1 \cdots dx_n}{(\cosh x_1 + \cdots + \cosh x_n)^2}.$$ 

We know this exactly for $n = 1, 2, 3, 4$; alas, no further! Happily, there is a **one-dimensional** representation

$$C_n = \frac{2^n}{n!} \int_0^\infty pK_0^n(p) \, dp,$$

This 1-dimensional form enabled us to perform extreme-precision quadrature on the $C$ integrals.
\[ C_{1024} = 0.63047350337438679612204019271087890435458707871273234157381798370897000382995819110189954165781719099450136225650411661308404743188411243430397157807755468454007309617205086544336866559818098035827274476038611125814904820814149091790648796301483682260404530555672606139009414570030164542749891640788518827356231464551258312731923493382586999271101529660669315266992303756802098645329501890289335012008…\]

We then sent 16–digits of this extreme-precision number to the CECM Inverse Symbolic Calculator facility at SFU and received the report…

Mixed constants, 2 with elementary transforms.
\[ 6304735033743867 = sr(2)^2/exp(gamma)^2 \]
A Fine Surprise!

\[ C_{1024} \sim 2 \, e^{-2\gamma} \quad \text{AGREES TO 300 PLACES} \]

Where \( \gamma \) is the celebrated Euler constant.

We could then prove much more than:

\[
C_n \sim 2 \, e^{-2\gamma} + \frac{n + 4}{2^n} e^{-4\gamma} \\
+ \frac{2n^2 + 23n + 57}{3^n \cdot 6} e^{-6\gamma} + \ldots
\]

- this gives \( C_{32} \) to 17 places
We ponder the “box integral” in $n$ dimensions:

\[ B_n := \int_0^1 \cdots \int_0^1 \sqrt{x_1^2 + x_2^2 + \ldots} \, dx_1 \, dx_2 \cdots dx_n. \]

This the expectation of radius from the origin, over the unit $n$-cube.

Trivially, but interestingly, the expected radius from the center of the $n$-cube is just $B_n / 2$. 
What is known about the $B_n$:

$B_1 = \frac{1}{2}$, $= 0.5$

$B_2 = \frac{\sqrt{2}}{3} + \frac{1}{3} \log(\sqrt{2} + 1)$, $= 0.765196…$

$B_3 = \frac{\sqrt{3}}{4} + \frac{1}{2} \log(2 + \sqrt{3}) - \frac{\pi}{24}$, $= 0.960592…$

$B_4 = -\frac{7}{60} \pi \sqrt{2} - \frac{1}{20} \pi \log(1 + \sqrt{2}) + \log 3 + \frac{2}{5}$

$-\frac{14}{15} \sqrt{2} \tan^{-1}(\sqrt{2}/4) + \frac{7}{15} \sqrt{2} \tan^{-1}(\sqrt{2}) + \frac{1}{10} J$.

$J := \int_0^1 \frac{\log(1 + \sqrt{3} + y^2) - \log(-1 + \sqrt{3} + y^2)}{1 + y^2} \, dy$ (hard integral)
One-dimensional Quadrature Again!

\[ B_n = \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{dt}{t^2} \left( 1 - \left( \frac{\sqrt{\pi} \operatorname{erf}(t)}{2t} \right)^n \right). \]

This allowed us to calculate box integrals to 1000’s of digits, even for large \( n \), suggesting for example the proven theorem

\[ B_n \sim \sqrt{\frac{n}{3}}. \]

Still, exact evaluations for \( n = 4, 5, \ldots \) remain elusive.

- better qMC integration is needed
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My Maple Wish List  (in order)

- LARGE SCALE PARALLEL MAPLE
  - Better numerical integration in 1D – ‘tanh-sinh’
  - Somewhat better numerical integration in 2D/3D
    - better use of ‘assuming’
  - More knowledge of analytic closed forms
    - Dirichlet L-series etc
  - Broader access to Real universe computation
    - especially with logs
  - Some connectivity to DLMF
  - Ability to call external interfaces
    - e.g., revamped Inverse Calculator, Sloane, MZV’s (EZ-face)
University – Industry links

MITACS – MRI
putting high end science on a hand held
Bringing Math Concepts to Life at Robert Morris College

by Dawn Henwood

It's just another Wednesday morning in a small applied math class in Chicago's Robert Morris College, but instructor Ed Clark is conscious that he's at the epicenter of an educational revolution. Clustered in small groups, Clark's students are engaged in a hands-on analysis of two competitive cell phone plans. Because all of the students have in hand a Dell Axim with MRI Graphing Calculator software, they're able to tackle the problem at their own pace and in their own way. With this powerful combination of hardware and software, Clark has transformed his classroom into an active mathematics "laboratory."

Clark and his colleagues have been working The effect of the new technology on Clark's teaching style has been dramatic. Previously he used up to a third of his class time just explaining how to work the calculator and guiding students step by step through complicated keystrokes. Now he focuses entirely on how to work the problems: he's free to engage students in what he calls "discovery learning." In some cases, he's able to cover a concept twice as quickly as it would have taken in the past.

Clark says that MRI Graphing Calculator and Pocket PCs have sharpened the focus of his teaching. "Just the fact that a handheld computer displays colors is huge," he notes, "especially when you are working with a problem that involves plotting and compar-
MRI’s First Product in Mid-nineties

Built on Harper Collins college dictionary - an IP adventure!

Maple inside the MathResource

Database now in Maple 9.5/10

CONVERGENCE?
A plot of 

\[ r = 1.3 \sin(p) \]

in spherical coordinates

The surface 

\[ z = \sin(x) + \cos(y) \]
anticlastic

adj. (of a surface) having curvatures of opposite signs in two perpendicular directions at a given point; saddle-shaped. For example, shown in the figure.

X is a minimum between A and B, but a maximum between C and D. Compare synclastic. See also saddle point.

➢ Any blue is a hyperlink
➢ Any green opens a reusable Maple window with initial parameters set
➢ Allows exploration with no learning curve
Using Portrait 4

The screen also contains number and function keys which are standard on most handheld calculators. To use any of these buttons simply tap on them.

The Plots menu is a part of the Functions menu. The trigonometric, hyperbolic, exponential and log functions are also found in the Functions menu.
PROTOTYPE for handheld collaboration

Industrial strength hardware and software throughout
"Just a darn minute! — Yesterday you said that $x$ equals two!"
Remote Visualization via Access Grid

• The touch sensitive interactive D-DRIVE
• Immersion & haptics
• and the 3D GeoWall

The future is here…

… just not uniformly
This picture is worth 100,000 ENIACs

The number of ENIACS needed to store the 20Mb TIF file the Smithsonian sold me

The past
NERSC's 6000 cpu Seaborg in 2004 (10Tflops/sec)
- we need new software paradigms for 'bigga-scale' hardware

The present

Mathematical Immersive Reality
in Vancouver
Supercomputer doubles own record

The Blue Gene/L supercomputer has broken its own record to achieve more than double the number of calculations it can do a second.

It reached 280.6 teraflops - that is 280.6 trillion calculations a second.

IBM BlueGene/L system at LLNL

System
(64 cabinets, 64x32x32)

2.8/5.6 GF/s 5.6/11.2 GF/s
4 MB 0.5 GB DDR

217 cpu’s

Oct 2005 It has now run Linpack benchmark at over 280 Tflop /sec (4 x Canadian-REN)
Moore’s 1965 Law continues:
But can we rely on Kurzweil’s projected ‘Singularity’?
The LRP tells a Story

• The Story
• Executive Summary
• Main Chapters
  – Technology
  – Operations
  – HQP
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One Day...

High-performance computing (HPC) affects the lives of Canadians every day. We can best explain this by telling you a story. It's about an ordinary family on an ordinary day, Russ, Susan, and Kerri Sheppard. They live on a farm 15 kilometres outside Wyoming, Ontario. The land first produced oil, and now it yields milk; and that's just fine locally.

Their day, Thursday, May 29, 2003, begins at 4:30 am when the alarm goes off. A busy day, Susan Zhong- Sheppard will fly to Toronto to see her father, Wu Zhong, at Toronto General Hospital; he's very sick from a stroke. She takes a quick shower and packs a day bag for her 6 am flight from Sarnia's Chris Hadfield airport. Russ Sheppard will stay home at their dairy farm, but his day always starts early. Their young daughter Kerri can sleep three more hours until school.

Waiting, Russ looks outside and thinks, It's been a dryish spring. Where's the rain?

In their farmhouse kitchen on a family-sized table sits a PC with a high-speed Internet line. He logs on and finds the Farmer Daily site. He then chooses the Environment Canada link, clicks on Ontario, and then scans down for Sarnia-Lambton.

WEATHER PREDICTION

The “quality” of a five-day forecast in the year 2003 was equivalent to that of a 36-hour forecast in 1963 [REF 1]. The quality of daily forecasts has risen sharply by roughly one day per decade of research and HPC progress. Accurate forecasts transform into billions of dollars saved annually in agriculture and in natural disasters. Using a model developed at Dalhousie University (Prof. Keith Thompson), the Meteorological Service of Canada has recently been able to predict coastal flooding in Atlantic Canada early enough for the residents to take preventative action.
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``The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it.``