“I feel so strongly about the wrongness of reading a lecture that my language may seem immoderate .... The spoken word and the written word are quite different arts .... I feel that to collect an audience and then read one’s material is like inviting a friend to go for a walk and asking him not to mind if you go alongside him in your car.”

Sir Lawrence Bragg

What would he say about .ppt?
ABSTRACT. Current and expected advances in computation and storage, collaborative environments and visualization make it possible to interact at a distance in many varied and flexible ways. I'll illustrate some of the present and emerging opportunities to share research and data, seminars, classes, planning meetings and much else fully at a distance.


**Challenges of MKM (Math Knowledge Management)**
- integration of tools, inter-operability
- e.g., workable mathematical OCR
- intelligent-agents, automated use
- many IP/copyright and sociological issues
- metadata, standards and on [www.mkm-ig.org](http://www.mkm-ig.org)
Remote Visualization via Access Grid

- The touch sensitive interactive D-DRIVE
- Immersion & haptics
- and the 3D GeoWall

The future is here…

… just not uniformly
<table>
<thead>
<tr>
<th>Microprocessor</th>
<th>Year of Introduction</th>
<th>Transistors</th>
</tr>
</thead>
<tbody>
<tr>
<td>4004</td>
<td>1971</td>
<td>2,300</td>
</tr>
<tr>
<td>8008</td>
<td>1972</td>
<td>2,500</td>
</tr>
<tr>
<td>8080</td>
<td>1974</td>
<td>4,500</td>
</tr>
<tr>
<td>8086</td>
<td>1978</td>
<td>29,000</td>
</tr>
<tr>
<td>Intel286</td>
<td>1982</td>
<td>134,000</td>
</tr>
<tr>
<td>Intel386™ processor</td>
<td>1985</td>
<td>275,000</td>
</tr>
<tr>
<td>Intel486™ processor</td>
<td>1989</td>
<td>1,200,000</td>
</tr>
<tr>
<td>Intel® Pentium® processor</td>
<td>1993</td>
<td>3,100,000</td>
</tr>
<tr>
<td>Intel® Pentium® II processor</td>
<td>1997</td>
<td>7,500,000</td>
</tr>
<tr>
<td>Intel® Pentium® III processor</td>
<td>1999</td>
<td>9,500,000</td>
</tr>
<tr>
<td>Intel® Pentium® 4 processor</td>
<td>2000</td>
<td>42,000,000</td>
</tr>
<tr>
<td>Intel® Itanium® processor</td>
<td>2001</td>
<td>25,000,000</td>
</tr>
<tr>
<td>Intel® Itanium® 2 processor</td>
<td>2003</td>
<td>220,000,000</td>
</tr>
<tr>
<td>Intel® Itanium® 2 processor (9MB cache)</td>
<td>2004</td>
<td>592,000,000</td>
</tr>
</tbody>
</table>

Moore’s Law in 1965 and 2005.
A. Communication, Collaboration and Computation.


B3. Inverse Symbolic Computation.

The talk ends when I do.

Much is still driven by particle physics and Moore’s Law (and soon biology):

- **AccessGrid**
- **User controlled light paths**
- **Atlas** (LHC hunt for the Higgs Boson)
  - TRIUMF using 1000 cpu, 1Peta-byte
- **Genomics and proteomics**
  - SARS decoded at Michael Smith Genome Centre
East meets West: Collaboration goes National

Welcome to D-DRIVE whose mandate is to study and develop resources specific to (‘dis-located’) distributed research and interaction in the sciences with first client groups being the following communities:

- High Performance Computing
- Mathematical and Computational Science Research
- Science Outreach
  - Research
  - Education
  - Media
D-DRIVE  Jon Borwein  P. Borwein (SFU)  D. Bailey (Lawrence Berkeley)
R. Crandall (Reed and Apple) and many others

Staff  David Langstroth (Manager)  Scott Wilson (Systems)
       Nolan Zhang (SysOp)  Peter Dobscanyi (HPC)

Students  Macklem (Parallel Optimization)  Wiersma (Analysis/NIST)
         Hamilton (Inequalities and Computer Algebra)  Ye (Quadrature)
        Paek (Federated search),  Oram (Haptics), et al

AIM: (Secure, Stable, Satisfying) Presence at a Distance
Content Provider: putting math and science on handhelds, laptops, web, in classrooms (LORs and authoring tools) …
MRI’s First Product in Mid-nineties

- Built on Harper Collins college dictionary - an IP adventure!
- Maple inside the MathResource
- Database now in Maple 9.5/10

MathResources Inc.

✓ often a 10 year lag between R&D and product
Bringing Math Concepts to Life at Robert Morris College

by Dawn Henwood

It’s just another Wednesday morning in a small applied math class in Chicago’s Robert Morris College, but instructor Ed Clark is conscious that he’s at the epicenter of an educational revolution. Clustered in small groups, Clark’s students are engaged in a hands-on analysis of two competitive cell phone plans. Because all of the students have in hand a Dell Axim with MRI Graphing Calculator software, they’re able to tackle the problem at their own pace and in their own way. With this powerful combination of hardware and software, Clark has transformed his classroom into an active mathematics “laboratory.”

Clark and his colleagues have been working on the new technology for several years. The effect of the new technology on Clark’s teaching style has been dramatic. Previously he used up to a third of his class time just explaining how to work the calculator and guiding students step by step through complicated keystrokes. Now he focuses entirely on how to work the problems: he’s free to engage students in what he calls “discovery learning.” In some cases, he’s able to cover a concept twice as quickly as it would have taken in the past.

Clark says that MRI Graphing Calculator and Pocket PCs have sharpened the focus of his teaching. “Just the fact that a handheld computer displays colors is huge,” he notes, “especially when you are working with a problem that involves plotting and compar-
PROTOTYPE for handheld collaboration

Industrial strength hardware and software throughout
"It says it's sick of doing things like inventories and payrolls, and it wants to make some breakthroughs in astrophysics."
The reader who wants to get an introduction to this exciting approach to doing mathematics can do no better than these books.
— Notices of the AMS

I do not think that I have had the good fortune to read two more entertaining and informative mathematics texts.
— Australian Mathematical Society Gazette

This Experiments in Mathematics CD contains the full text of both Mathematics by Experiment: Plausible Reasoning in the 21st Century and Experimentation in Mathematics: Computational Paths to Discovery in electronic, searchable form. The CD includes several "smart" enhancements, such as:

- Hyperlinks for all cross references
- Hyperlinks for all Internet URLs
- Hyperlinks to bibliographic references
- Enhanced search function, which assists one with a search for a particular mathematical formula or expression.

These enhancements significantly improve the usability of these files and the reader's experience with the material.
Experimental Methodology

1. Gaining insight and intuition
2. Discovering new relationships
3. Visualizing math principles
4. Testing and especially falsifying conjectures
5. Exploring a possible result to see if it merits formal proof
6. Suggesting approaches for formal proof
7. Computing replacing lengthy hand derivations
8. Confirming analytically derived results

Many people regard mathematics as the crown jewel of the sciences. Yet math has historically lacked one of the defining trappings of science: laboratory equipment. Physicists have their particle accelerators; biologists, their electron microscopes; and astronomers, their telescopes. Mathematicians, by contrast, concern not the physical landscape but an理想信念, abstract world. For exploring that world, mathematicians have traditionally had only their imaginations.

New computers are starting to give mathematicians the laboratory equipment they have been missing. Symbolic mathematics software is enabling researchers to travel farther and deeper into the mathematical universe. They are extending the number fields with high-precision calculations, for instance, or discovering patterns in the contours of beautiful, infinite families of curves that arise out of the geometry of shapes. Mathematicians in the computer lab are leading mathematicians to discover and understand truths that had never reached their traditional models. "Pretty much every (mathematical) field has been transformed," says Richard Courant, a mathematician at Rice College in Portland, Ore. "Instead of just being a number crunching tool, the computer is becoming more like a general theory that moves around, and you find incredible results!"

As the same time, the new tools are opening a few questions about how to extend experimental results.

Comparing $-y^2 \ln(y)$ (red) to $y - y^2$ and $y^2 - y^4$
Components include

- AccessGrid
- UCLP for
  - haptics
  - learning objects
  - visualization
- Grid Computing
- Data Mining
2003: Me and my Avatar
- designer now works for William Shatner (‘Wild’)
Coast to Coast Seminar Series (C2C)

Lead partners:

Dalhousie D-Drive – Halifax
Nova Scotia

IRMACS – Burnaby, British Columbia

Other Participants so far:

University of British Columbia, University of Alberta, University of Alberta University of Saskatchewan, Lethbridge University, Acadia University, St Francis Xavier University, University of Western Michigan, MathResources Inc, University of North Carolina

Tuesdays 3:30 – 4:30 pm Atlantic Time

http://projects.cs.dal.ca/ddrive/
The Experience

Fully Interactive multi-way audio and video

audio is harder (given good bandwidth)

The closest thing to being in the same room

Shared Desktop for viewing presentations or sharing software
The 2,500 sq-metre IRMACS research centre

The $14 million centre’s acronym stands for interdisciplinary research in the mathematical and computational sciences. The centre’s expansive view of the world and from atop a mountain echoes its potential as a facility fostering frontier research.

SFU building is a also a 190cpu G5 Grid

At the official April 2005 opening, I gave one of the four presentations from D-DRIVE.

Trans-Canada ‘C2C’ Seminar
Tuesdays PST 11.30 MST 12.30 AST
3.30 and even 7.30 GMT
[March 30 – from MRInc]
Jonathan Schaeffer, University of Edmonton
Solving Checkers

Przemyslaw Prusinkiewicz, University of Calgary
Computational Biology of Plants

Jonathan Borwein, Dalhousie University
Mathematical Visualization

Uwe Glaesser, Simon Fraser University
Semantic Blueprints of Discrete Dynamic Systems

Peter Borwein, IRMACS
The Riemann Hypothesis

Arvind Gupta, MITACS
The Protein Folding Problem

‘No one explains chalk’

Karl Dilcher, Dalhousie University
Fermat Numbers, Wieferich and Wilson Primes

High Quality Presentations
Haptics in the MLP

Haptic Devices extend the world of I/O into the tangible and tactile

We link multiple devices so two or more users may interact at a distance (BC/NS Demo April 06)

- in Museums and elsewhere
- Kinesiology, HCI, Surgery …
My remaining intention is to show a variety of mathematical uses of high performance computing and communicating as part of Experimental Inductive Mathematics.

“Elsewhere Kronecker said "In mathematics, I recognize true scientific value only in concrete mathematical truths, or to put it more pointedly, only in mathematical formulas." ... I would rather say "computations" than "formulas", but my view is essentially the same.”

Harold Edwards, Essays in Constructive Mathematics, 2004
Very cool for the one person with control
"What I appreciate even more than its remarkable speed and accuracy are the words of understanding and compassion I get from it."
Outline of ACE Talk

A. Communication, Collaboration and Computation.
B3. Inverse Symbolic Computation.

The talk ends when I do.
COXETER’S (1927) Kaleidoscope
The Perko Pair $10_{161}$ and $10_{162}$

are two adjacent 10-crossing knots (1900)

• first shown to be the same by Ken Perko in 1974
• and beautifully made dynamic in KnotPlot (open source)
An unusual Mandelbrot parameterization

Various visual examples follow

- Indra’s pearls
- Roots of `1/-1’ polynomials
- Ramanujan’s fraction

AK Peters, 2004
(CD, 2006)
Double cusp group

2002: http://klein.math.okstate.edu/IndrasPearls/
FOUR DEMOS combining inversion, reflection and dilation

1. Indraspearsls
2. Apollonius*
3. Hyperbolicity
4. Gasket
Striking fractal patterns formed by plotting complex zeros for all polynomials in powers of $x$ with coefficients 1 and -1 to degree 18

Coloration is by sensitivity of polynomials to slight variation around the values of the zeros. The color scale represents a normalized sensitivity to the range of values; red is insensitive to violet which is strongly sensitive.

- All zeros are pictured (at 3600 dpi)
- Figure 1b is colored by their local density
- Figure 1d shows sensitivity relative to the $x^9$ term
- The white and orange striations are not understood

A wide variety of patterns and features become visible, leading researchers to totally unexpected mathematical results

"The idea that we could make biology mathematical, I think, perhaps is not working, but what is happening, strangely enough, is that maybe mathematics will become biological!"

The TIFF on VARIOUS SCALES

Pictures are more democratic but they come from formulae
Roots in the most stable colouring
Why should I refuse a good dinner simply because I don't understand the digestive processes involved?

Oliver Heaviside (1850 - 1925)

- when criticized for his daring use of operators before they could be justified formally
Numeric and Symbolic Computation

- Central to my work - with Dave Bailey - meshed with visualization, randomized checks, many web interfaces/databases (NIST)
- Massive (serial) Symbolic Computation
  - Automatic differentiation code
- Integer Relation Methods
- Inverse Symbolic Computation

Other useful tools: Parallel Maple
- Sloane’s online sequence database
- Salvy and Zimmerman’s generating function package ‘gfun’
- Automatic identity proving: Wilf-Zeilberger method for hypergeometric functions
Maple on SFU 192 cpu ‘bugaboo’ cluster
2002 - different node sets are in different colors
§Al.4. Maclaurin Series

For $z \in \mathbb{C}$

Al4.1

$A_l (z) = A_l (0) \left( 1 + \frac{1}{3!} z^3 + \frac{1}{6!} z^6 + \frac{1}{9!} z^9 + \cdots \right) + A_l' (0) \left( z + \frac{2}{4!} z^4 + \frac{2.5}{7!} z^7 + \frac{2.5.8}{10!} z^{10} + \cdots \right)$

Symbols used:
- AiryAi
- cdots
- and

A&S Ref:
- 10.4.2 (with 10.4.4 and 10.4.5)

Encodings:
- LaTeX
- Parsed:

$\text{Al4.2}$

$A_l (z) = A_l (0) \left( 1 + \frac{1}{3!} z^3 + \frac{2.5}{6!} z^6 + \frac{2.5.8}{9!} z^9 + \cdots \right) + A_l' (0) \left( \frac{1}{2!} z^2 + \frac{1.4.5}{3!} z^5 + \frac{1.4.7}{8!} z^8 + \cdots \right)$

$\text{Al4.3}$

$B_i (z) = B_i (0) \left( 1 + \frac{1}{3!} z^3 + \frac{1.4.7}{6!} z^6 + \frac{1.4.7}{9!} z^9 + \cdots \right) + B_i' (0) \left( z + \frac{2}{4!} z^4 + \frac{2.5}{7!} z^7 + \frac{2.5.8}{10!} z^{10} + \cdots \right)$

$\text{Al4.4}$

$B_i' (z) = B_i' (0) \left( 1 + \frac{1}{3!} z^3 + \frac{2.5}{6!} z^6 + \frac{2.5.8}{9!} z^9 + \cdots \right) + B_i (0) \left( \frac{1}{2!} z^2 + \frac{1.4.5}{3!} z^5 + \frac{1.4.7}{8!} z^8 + \cdots \right)$
"What it comes down to is our software is too hard and our hardware is too soft."
A. Communication, Collaboration and Computation.


B3. Inverse Symbolic Computation.

The talk ends when I do.
Let \((x_n)\) be a vector of real numbers. An integer relation algorithm finds integers \((a_n)\) such that
\[a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0\]

- At the present time, the PSLQ algorithm of mathematician-sculptor Helaman Ferguson is the best-known integer relation algorithm.
- PSLQ was named one of ten “algorithms of the century” by Computing in Science and Engineering.
- High precision arithmetic software is required: at least \(d\times n\) digits, where \(d\) is the size (in digits) of the largest of the integers \(a_k\).

**An Immediate Use**

To see if \(a\) is algebraic of degree \(N\), consider \((1, a, a^2, \ldots, a^N)\)
Peter Borwein in front of Helaman Ferguson’s work

CMS Meeting
December 2003
SFU Harbour Centre

Ferguson uses high tech tools and micro engineering at NIST to build monumental math sculptures
\[ \zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \]

\[ = \frac{\pi^2}{6}, \zeta(4) = \frac{\pi^4}{90}, \zeta(6) = \frac{\pi^6}{945}, \ldots \]

1. via PSLQ to 50,000 digits (250 terms)

\[ \mathcal{Z}(x) = \sum_{k=0}^{\infty} \zeta(2k+2)x^{2k} = \sum_{n=1}^{\infty} \frac{1}{n^2 - x^2} \]

2. reduced as hoped

\[
3n^2 \sum_{k=n+1}^{2n} \frac{\prod_{m=n+1}^{k-1} \left( \frac{1}{k} \right) \left( \frac{4n^2-m^2}{n^2-m^2} \right)}{k^2-n^2} = \frac{1}{2n} - \frac{1}{3n} \]

3. was easily computer proven (Wilf-Zeilberger) human proof (MAA)?
My brother made the observation that this log formula allows one to compute binary digits of log 2 without knowing the previous ones! (a BBP formula)

This reduced to

\[ \log 2 = \sum_{n=1}^{\infty} \frac{1}{k 2^k} \]

Maple, Mathematica and humans can easily prove. A triumph for "reverse engineered mathematics" algorithm design.

Bailey, Plouffe and he hunted for such a formula for Pi. Three months later the computer - doing bootstrapped PSLQ hunts - returned:

No such formula exists base-ten (provably)

Finalist for the $100K Edge of Computation Prize won by David Deutsch.

---

**Edge The Third Culture**

---

**THE $100,000 EDGE OF COMPUTATION SCIENCE PRIZE**

For individual scientific work, extending the computational idea, performed, published, or newly applied within the past ten years.

The Edge of Computation Science Prize, established by Edge Foundation, Inc., is a $100,000 prize initiated and funded by science philanthropist Jeffrey Epstein.
IF THERE WERE COMPUTERS IN GALILEO'S TIME
Outline of HPM Talk

A. Communication, Collaboration and Computation.


B3. Inverse Symbolic Computation.

The talk ends when I do.
Inverse Symbolic Computation

Inferring mathematical structure from numerical data

Mixes large table lookup, integer relation methods and intelligent preprocessing – needs micro-parallelism

- It faces the “curse of exponentiality”
- Implemented as Recognize in Mathematica and identify in Maple

Expressions that are not numeric like \( \ln(\pi \sqrt{2}) \) are evaluated in Maple in symbolic form first, followed by a floating point evaluation followed by a lookup.
Donald Knuth asked for a closed form evaluation of:

\[
\sum_{k=1}^{\infty} \left\{ \frac{k^k}{k! e^k} - \frac{1}{\sqrt{2\pi k}} \right\} = -0.084069508727655 \ldots
\]

The answer is Gonnet's Lambert's W which solves

\[ W \exp(W) = x \]

It's easy to compute 20 or 200 digits.

We thus have a prediction which Maple 9.5 on a laptop confirms to 100 places in under 6 seconds and to 500 in 40 seconds.

* ARGUABLY WE ARE DONE
ENTERING

evalf(Sum(k^k/k!/exp(k)-1/sqrt(2*Pi*k),k=1..infinity),16) = K

'Simple Lookup' fails; 'Smart Lookup' gives:

Results of the search:

Maple output:

.08406950872765600e-1

Value to be looked up: .08406950872765600e-1

Performing a smart lookup on .08406950872765600e-1:

<table>
<thead>
<tr>
<th>Function</th>
<th>Result</th>
<th>Precision</th>
<th>Matches</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-2/3</td>
<td>0.82597157939010666666666</td>
<td>16</td>
<td>1</td>
</tr>
</tbody>
</table>

In the ISC, K was probably generated by one of the algorithms or found in one of the given tables. Answers are given from shortest to longest description.

Mixed constants with 5 operations:

5825971579390106 = zeta(1/2)/sin(2)/sin(Pi)

Browse around .5825971579390106.
Outline of ACE Talk

A. Communication, Collaboration and Computation.
B3. Inverse Symbolic Computation.
C. Computational Conclusion.

The talk ends when I do.
This picture is worth 100,000 ENIACs

The number of ENIACS needed to store the 20Mb TIF file the Smithsonian sold me

The past
NERSC’s 6000 cpu Seaborg in 2004 (10Tflops/sec)
- we need new software paradigms for ‘bigga-scale’ hardware

Mathematical Immersive Reality in Vancouver
Supercomputer doubles own record

The Blue Gene/L supercomputer has broken its own record to achieve more than double the number of calculations it can do a second.

It reached 280.6 teraflops - that is 280.6 trillion calculations a second.

2\textsuperscript{17} cpu’s

Oct 2005 It has now run Linpack benchmark at over 280 Tflop /sec (4 x Canadian-REN)
"Just a darn minute! — Yesterday you said that $X$ equals two!"
CONCLUSION

ENGINES OF DISCOVERY: The 21st Century Revolution
The Long Range Plan for High Performance Computing in Canada
The LRP tells a Story

One Day ...

High-performance computing (HPC) affects the lives of Canadians every day. We can best explain this by telling you a story. It’s about an ordinary family on an ordinary day, Russ, Susan, and Kerri Sheppard. They live on a farm 15 kilometres outside Wyoming, Ontario. The land first produced oil, and now it yields milk; and that’s just fine locally.

Their day, Thursday, May 29, 2003, begins at 4:30 am when the alarm goes off. A busy day, Susan Zhong-Sheppard will fly to Toronto to see her father, Wu Zhong, at Toronto General Hospital; he’s very sick from a stroke. She takes a quick shower and packs a day bag for her 6 am flight from Sarnia’s Chris Hadfield airport. Russ Sheppard will stay home at their dairy farm, but his day always starts early. Their young daughter Kerri can sleep three more hours until school.

Waiting, Russ looks outside and thinks, It’s been a dryish spring. Where’s the rain?

In their farmhouse kitchen on a family-sized table sits a PC with a high-speed Internet line. He logs on and finds the Farmer Daily site. He then chooses the Environment Canada link, clicks on Ontario, and then scans down for Sarnia-Lambton.

WEATHER PREDICTION

The “quality” of a five-day forecast in the year 2003 was equivalent to that of a 36-hour forecast in 1963 [REF 1]. The quality of daily forecasts has risen sharply by roughly one day per decade of research and HPC progress. Accurate forecasts transform into billions of dollars saved annually in agriculture and in natural disasters. Using a model developed at Dalhousie University (Prof. Keith Thompson), the Meteorological Service of Canada has recently been able to predict coastal flooding in Atlantic Canada early enough for the residents to take preventative action.
The backbone that makes so much of our Canadian science possible.


“The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it.”