What's New, What's Possible, What's Coming ...

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“intuition comes to us much earlier and with much less outside influence than formal arguments which we cannot really understand unless we have reached a relatively high level of logical experience and sophistication.”

George Polya 1887-1985

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Outline of Presentation

I. The **Changing Research Landscape**

II. New Ways of **Doing Mathematics**

III. New Ways of **Seeing Mathematics**

IV. Amazing New **Web Services**
I. Changing Research Landscape: a new triangle

Experimental
(wet science)

Computational
(dry science)

Theoretical
(thought experiments)
2008: a sustained Petaflop attained at LLL—2 years early

The array of Canadian research projects each have unique high performance computing requirements.

**Ring 1**
Desktop Computers / Small Clusters

**Ring 2**
Mid-Range Systems (in the top 500 worldwide)

**Ring 3**
Supercomputers / Terascale System (in the top 30 worldwide)
D-Drive's Nova Scotia location lends us unusual freedom when interacting globally. Many cities around the world are close enough in a chronological sense to comfortably accommodate real-time collaboration.
Moore’s Law and its Implications

“The complexity for minimum component costs has increased at a rate of roughly a factor of two per year ...

• now taken as “every 18 months to 2 years”

Certainly over the short term this rate can be expected to continue, if not to increase. Over the longer term, the rate of increase is a bit more uncertain, although there is no reason to believe it will not remain nearly constant for at least 10 years. That means by 1975, the number of components per integrated circuit for minimum cost will be 65,000. I believe that such a large circuit can be built on a single wafer.

Gordon Moore (Intel) "Cramming more components onto Electronic Circuits", *Electronics Magazine* 19 April 1965

Unprecedented and expected to continue for 10-20 years.
This picture is worth 100,000 ENIACs

The number of ENIACS needed to store the 20Mb TIF the Smithsonian sold me

1947 The past (5Kf/sec)
NERSC’s 6000 cpu Seaborg in 2004 (10Tflops/sec) - we need new software paradigms for `bigga-scale’ hardware
IBM Computer Achieves Petaflop Performance

6/9/2008

A National Nuclear Security Administration (NNSA) supercomputer has achieved an operational rate of 1,000 trillion calculations per second, or 1 petaflop, making the Roadrunner -- which the NNSA commissioned IBM Corp. to build in 2006 for around $130 million -- the world's fastest computer, the agency announced today.

The future 2005-2010

217 cpu's: Oct 2007 ran Linpack benchmark at over 596 Tflop/sec (5 x Canada or 8 x Oz)
Things we can’t model here include:

Self assembling wires 2nm apart (HP Labs)
"It says it's sick of doing things like inventories and payrolls, and it wants to make some breakthroughs in astrophysics."
II. New Ways of Doing Math

• and **related subjects**: Computer Science, Statistics, Engineering, all Sciences, *every other subject* ….* for learning or for research*
  
  – Experimentally on the Computer
  – Visual or Haptic or Acoustic Output
  – Simulations and Emersions
  – With Web-services, Databases, Wikis, …
    • Marvelous support tools for the classroom

• also **New Ways of Collaborating**
The following is a list of useful math tools.

**Utilities**
1. ISC2.0: The Inverse Symbolic Calculator
2. EZ Face: An interface for evaluation of Euler sums and Multiple Zeta Values
3. 3D Function Grapher
4. GraPHedron: Automated and computer assisted conjectures in graph theory
5. Julia and Mandelbrot Set Explorer
6. Embree-Trefethen-Wright pseudospectra and eigenproblem

**Reference**
7. The On-Line Encyclopedia of Integer Sequences
8. Finch’s Mathematical Constants
9. The Digital Library of Mathematical Functions
10. The Prime Pages

**Content**
11. Experimental Mathematics Website
12. Wolfram Mathworld
13. Planet Math
14. Numbers, Constants, and Computation
15. Wikipedia: Mathematics

**ICCOPT 2007 Short Course**
16. Jon's Lectures
17. [Link to ICCOPT 2007 Short Course](#)
Experimental Mathematics in Action

The last twenty years have witnessed a fundamental shift in the way mathematics is practiced. With the continued advance of computing power and accessibility, the view that “real mathematicians don’t compute” no longer has any traction for a newer generation of mathematicians that can really take advantage of computer-aided research, especially given the scope and availability of modern computational packages such as Maple, Mathematica, and MATLAB. The authors provide a coherent variety of accessible examples of modern mathematics subjects in which intelligent computing plays a significant role.

Advance Praise for Experimental Mathematics in Action

“Experimental mathematics has not only come of age but is quickly maturing as this book shows so clearly. The authors display a vast range of mathematical understanding and connection while at the same time delineating various ways in which experimental mathematics is and can be undertaken, with startling effect.”

—Prof. John Mason, Open University and University of Oxford

“Computing is to mathematics as telescope is to astronomy: it might not explain things, but it certainly shows ‘what’s out there.’ The authors are expert in the discovery of new mathematical ‘planets,’ and this book is a beautifully written exposition of their values, their methods, their subject, and their enthusiasm about it. A must read.”

—Prof. Herbert S. Wilf, author of generatingfunctionology

“From within the ideological blizzard of the young field of Experimental Mathematics comes this tremendous, clarifying book. The authors—all experts—convey this complex new subject in the best way possible, namely, by fine example. Let me put it this way: Discovering this book is akin to finding an emerald in a snowdrift.”

Math + Physics = Computing?

- En français
Haptics and Light Paths

Haptic Devices extend the world of I/O into the tangible and tactile

Links multiple devices so two or more users may interact at a distance (BC/NS Demo April 06)

- in Museums, Aware Homes, elsewhere
- Kinesiology, Surgery, Music, Art …
Very cool for the one person with control - and very expensive
Cost effective 3D visualization in 2007
19th C model plus recent photograph and 21st C rendition

Mathematical Form 0001
Helicoid: minimal surface.

\[
\begin{align*}
x &= a \sinh \nu \cos \mu \\
y &= a \sinh \nu \sin \mu \\
z &= au
\end{align*}
\]

\((0 \leq \mu < 2\pi, -\infty < \nu < \infty)\)
A second plaster model

19th C Plaster Model
Kline and Schwartz

Mathematical Form 0003

Hiroshi Sugimoto
Coast to Coast (‘C2C’) Seminar

2008: will focus on PhD presentations
Chile has now joined

Lead partners:
Dalhousie D-Drive – Halifax
Nova Scotia
IRMACS – Burnaby,
British Columbia

Other Participants so far include:
University of British Columbia, University of Alberta, University of Saskatchewan, Lethbridge University, Acadia University, MUN, Mt Allison, St Francis Xavier University, University of Western Michigan, MathResources Inc, University of North Carolina, …

Tuesdays 3:30pm (Atlantic) 11.30am (Pacific)
✓Chapter in Communicating Mathematics in the Digital Era (AK Peters, Sept 2008)
The Experience

Fully Interactive multi-way audio and visual interaction

Given good bandwidth audio is much harder

The closest thing to being in the same room

I could be in Newcastle AG

CARMA is coming

Computer Assisted Research Maths and its Applications

Shared Desktop for viewing presentations or sharing software
Content Dominates Form

High Quality Presentations

Jonathan Borwein, Dalhousie University
Mathematical Visualization

Uwe Glaesser, Simon Fraser University
Semantic Blueprints of Discrete Dynamic Systems

Peter Borwein, IRMACS
The Riemann Hypothesis

“No one explains chalk”

Jonathan Schaeffer, University of Alberta
Solving Checkers

Arvind Gupta, MITACS
The Protein Folding Problem

Przemyslaw Prusinkiewicz, University of Calgary
Computational Biology of Plants

Karl Dilcher, Dalhousie University
Fermat Numbers, Wieferich and Wilson Primes

Future Libraries will include very complex objects
“Solving Checkers”

Speaker in Edmonton

Audience in Vancouver

April 2007 Checkers solved

Science: one of top 10 break-throughs of 2007

2006: Poincaré Conjecture top breakthrough of year
III. New Ways of Seeing Math

• The Colour Calculator
  – numbers as pictures

• The Inverse Calculator
  – numbers go in and symbols come out

• The Top Ten Numbers Website

• All at http://ddrive.cs.dal.ca/~isc/portal
Inverse Symbolic Computation

Inferring mathematical structure from numerical data

- Mixes large table lookup, integer relation methods and intelligent preprocessing – needs micro-parallelism
- It faces the “curse of exponentiality”
- Implemented as identify in Maple

Expressions that are not numeric like ln(3)*sqrt(2)) are evaluated in Maple in symbolic form first, followed by a floating point evaluation followed by a lookup.
The Inverse Symbolic Calculator (ISC) uses a combination of lookup tables and integer relation algorithms in order to associate with a user-defined, truncated decimal expansion (represented as a floating point expression) a closed form representation for the real number.

Standard lookup results for $12.587886229548403854$

\[ \exp(1) + \pi^2 \]

$3.146264370$ Try it!

$19.99909998$ Try it!

- ISC+ runs on Glooscap
- Less lookup & more algorithms than 1995
Striking fractal patterns formed by plotting complex zeros for all polynomials in powers of $x$ with coefficients 1 and -1 to degree 18

Coloration is by sensitivity of polynomials to slight variation around the values of the zeros. **The color scale represents a normalized sensitivity** to the range of values; red is insensitive to violet which is strongly sensitive.

- All zeros are pictured (at 3600 dpi)
- Figure 1b is colored by their local density
- Figure 1d shows sensitivity relative to the $x^9$ term
- The white and orange striations are not understood

A wide variety of patterns and features become visible, leading researchers to totally unexpected mathematical results

"The idea that we could make biology mathematical, I think, perhaps is not working, but what is happening, strangely enough, is that maybe mathematics will become biological!"  
Greg Chaitin, [Interview](#), 2000.
Pictures are more democratic but they come from formulae
The Perko Pair $10_{161}$ and $10_{162}$ are two adjacent 10-crossing knots (1900)

- first shown to be the same by Ken Perko in 1974
- and beautifully made dynamic in KnotPlot (open source-ish)
"What it comes down to is our software is too hard and our hardware is too soft."
IV. Amazing New Web Services

- **AT&T** Online Encyclopedia of Sequences
  What is 1, 2, 3, 6, 11, 23, 47, 106, 235, ... ?

- **NIST** Digital Library of Math Functions
  What is an Airy Function?

- **MAA** Digital Library with my company’s free dictionary
  – also in *Maple* since 9.5

**Soon the texts will also do lots of the maths**

Supernumerary Rainbow over Newton’s birthplace
Greetings from the On-Line Encyclopedia of Integer Sequences!

Matches (up to a limit of 30) found for 1 2 3 6 11 23 47 106 235:
[It may take a few minutes to search the whole database, depending on how many matches are found (the second and later looks are faster)]

**An Exemplary Database**

- **ID Number:** A000055 (Formerly N0791 and N0299)
- **Sequence:** 1,1,1,1,2,3,6,11,23,47,106,235,551,1301,3159,7741,19320, 48629,123867,317995,823065,2144505,5623756,14828074, 39299897,104636890,279793450,751065460,2023443032, 5489566585,14630871802,40330829030,109997241022
- **Name:** Number of trees with n unlabeled nodes.
- **Comments:** Also, number of unlabeled 2-gonal 2-trees with n 2-gons.
- **References:**
  - N. L. Biggs et al., Graph Theory 1736-1936, Oxford, 1976, p. 49.
- **Links:**
  - P. J. Cameron, Sequences realized by oligomorphic permutation groups J. Integ. Seqs.
  - Steven Finch, Otter's Tree Enumeration Constants
  - E. Rains and N. J. A. Sloane, On Cayley's Enumeration of Alkanes (or 4-Valent Trees)
  - N. J. A. Sloane, Illustration of initial terms
  - E. W. Weisstein, Link to a section of The World of Mathematics.
  - Index entries for sequences related to trees
  - Index entries for "core" sequences
- **Formula:** $A(x) = 1 + T(x) - T(x)^2 + 2xT(x) + 2xT(x^2)/2$, where $T(x) = x + x^2 + 2x^3 + ...$

Integrated real time use

- **142,759 entries**
- **grows daily**
- **AP book had 5,000**
§A1.4. Maclaurin Series

For $z \in \mathbb{C}$

A1.4.1

$A_i(z) = A_i(0) \left( 1 + \frac{1}{3!} z^3 + \frac{1}{5!} z^5 + \frac{1}{7!} z^7 + \cdots \right) + A_i'(0) \left( \frac{z}{2!} z^2 + \frac{2}{4!} z^4 + \frac{2}{6!} z^6 + \frac{2}{8!} z^8 + \cdots \right) + \cdots$

Symbols used:
- $A_i$, $A_i'$, $cdots$, and $z$

A&S Ref:
- 10.4.2 (with 10.4.4 and 10.4.5)

Encodings:
- LaTeX
- Parsed:
  - $A_i(z) = A_i(0) \left( 1 + \frac{z}{3!} z^3 + \frac{z^5}{5!} z^5 + \frac{z^7}{7!} z^7 + \cdots \right) + A_i'(0) \left( \frac{z}{2!} z^2 + \frac{2 z^4}{4!} z^4 + \frac{2 z^6}{6!} z^6 + \frac{2 z^8}{8!} z^8 + \cdots \right) + \cdots$

- $B_i(z) = B_i(0) \left( 1 + \frac{z}{3!} z^3 + \frac{1}{5!} z^5 + \frac{1}{7!} z^7 + \cdots \right) + B_i'(0) \left( \frac{z}{2!} z^2 + \frac{2 z^4}{4!} z^4 + \frac{2 z^6}{6!} z^6 + \frac{2 z^8}{8!} z^8 + \cdots \right) + \cdots$

A1.4.2

$A_i'(z) = A_i'(0) \left( 1 + \frac{z}{3!} z^3 + \frac{2}{5!} z^5 + \frac{2}{7!} z^7 + \cdots \right) + A_i(0) \left( \frac{z}{2!} z^2 + \frac{1}{4!} z^4 + \frac{1}{6!} z^6 + \cdots \right)$

A1.4.3

$B_i(z) = B_i(0) \left( 1 + \frac{z}{3!} z^3 + \frac{1}{5!} z^5 + \frac{1}{7!} z^7 + \cdots \right) + B_i'(0) \left( \frac{z}{2!} z^2 + \frac{2}{4!} z^4 + \frac{2}{6!} z^6 + \frac{2}{8!} z^8 + \cdots \right)$

A1.4.4

$B_i'(z) = B_i'(0) \left( 1 + \frac{z}{3!} z^3 + \frac{2}{5!} z^5 + \frac{2}{7!} z^7 + \cdots \right) + B_i(0) \left( \frac{z}{2!} z^2 + \frac{1}{4!} z^4 + \frac{1}{6!} z^6 + \cdots \right)$
The faint line below the main colored arc is a 'supernumerary rainbow', produced by the interference of different sun-rays traversing a raindrop and emerging in the same direction. For each color, the intensity profile across the rainbow is an Airy function. Airy invented his function in 1838 precisely to describe this phenomenon more accurately than Young had done in 1800 when pointing out that supernumerary rainbows require the wave theory of light and are impossible to explain with Newton's picture of light as a stream of independent corpuscles. The house in the picture is Newton's birthplace.
IF THERE WERE COMPUTERS IN GALILEO’S TIME
REFERENCES


“The object of mathematical rigor is to sanction and legitimize the conquests of intuition, and there was never any other object for it.”

Karl Heinz Hoffmann’s Cover Illustrations for

The Computer as Crucible