Modelling Wave Propagation, Scattering, and Attenuation in the Marginal Ice Zone

Michael Meylan

Department of Mathematics, University of Auckland
Outline

1. The Marginal Ice Zone
2. Measuring Wave Processes
3. Wave-Ice Interaction Models
4. Elastic Plate Model
5. Modelling the MIZ
6. Conclusions
The Marginal Ice Zone

- What is the Marginal Ice Zone?
- What role does it play in the global climate?
- What role does wave action play in determining its structure?
Figure: Antarctic
Formation of Sea Ice

- The formation of sea ice is more complicated than for fresh water ice.
- Ice generally passes through frazil ice, nilas ice and pancake ice on its way to forming a continuous ice cover.
- This leads to a complicated structure.
Figure: Grease Ice
Figure: Pancake Ice
Figure: Pancake Ice
Figure: Continuous Ice Cover
Figure: Airport in Antarctic
Edge Processes

- The pack ice is subject to intense wave forces at the ice edge.
- These forces break the ice so that at the boundary of open and frozen ocean an interfacial region, the marginal ice zone, forms.
Figure: Ice Edge
Figure: Marginal Ice Zone
Figure: Marginal Ice Zone
Figure: Break up
Figure: Break up
Figure: Break up
Figure: Sunset
Measuring Wave Processes

- Polar regions are extremely demanding places to conduct research.
- A series of wave measurements were made during the 1980s using wave buoys (mostly by Wadhams et. al at the Scott Polar Research Institute).
- Many people have tried to make measurements of wave processes using satellites but unfortunately this has not proved successful.
- Best results were obtained by Squire and Moore 1980.
Figure: Wave Attenuation Experiment
Figure: Experimental Results
Figure 1. Attenuation rates of ocean waves damped by sea ice as reported by Wadhams et al. [1988] (compare equation (1)). The estimates were obtained under various ice conditions in the Greenland Sea and the Bering Sea.

Figure: Summary of Results of Wadhams et. al 1988
Modelling Wave-Ice Interaction

- All models assume that the wave processes in the ocean are linear.
- Models are proposed for different conditions and have different assumptions.
- No established best model.
- Large uncertainty in sea ice parameters
Mass Loading Model

- Assumes that sea ice simple is a mass floating on the water surface.
- This mass changes the propagation speed.
- Cannot account for anything more than edge effects, i.e. cannot explain the attenuation of waves through ice.
- Original paper was by Weitz and Keller 1950.
- Has proved to be reasonably successful for wave propagation through frazil or pancake ice.
Eddy Viscosity Model

- Model was proposed by Weber 1987.
- The system is modelled by a two layer fluid, with the ice represented by a viscous Newtonian fluid.
- It is best suited to model frazil or pancake ice but has been applied to large floes as well.
- Works well, but contains a free parameter, the eddy viscosity, for which there is no independent way of determining.
Floe Collision Model

- Model based on simulating floating ice and allowing collisions.
- Applies best to small floes and to energetic wave processes.
- Difficult to know the values of many of the parameters, such as coefficient of restitution.
- Requires extensive numerical simulation to give any results.
Rigid Body Model

- Ice floes are assumed rigid.
- This allows standard ship seakeeping methods to determine the wave scattering.
- Used by Masson and LeBlond 1989, who also proposed a method to overcome difficult problem of moving from small to large length scales.
- Model was applied practically by Perrie and Hu 1996, who obtained reasonable agreement with data.
Elastic Plate Model

- Long history in ice going back more than 100 years.
- Large floes (wavelength sized) have been shown to bend significantly.
- Simplified solution found by Wadhams 1986 which fitted (roughly) with measurements.
- Publication of the equations has lead to strong interest from applied mathematicians, independently of geophysical considerations.
- I will focus on the elastic plate model for the remainder of the talk.
Equations for the Elastic Plate Model

\[ \frac{\partial \phi}{\partial z} = \frac{\omega^2}{g} \phi \]

\[ \nabla^2 \phi = 0 \]

\[ \frac{\partial \phi}{\partial z} = 0 \quad z = H \]

Figure: Elastic Plate Model for an Ice Floe
Ice Floe Equation

- We couple the equation for a thin plate with the pressure due to the water.

\[ D \nabla^4 W - \omega^2 \phi_i h W = \rho g W - i \omega \phi \]

- Where \( W \) is the displacement of the plate and \( \phi \) is the velocity potential of the water.

- We must also include the boundary conditions at the edge of the plate.

- Generally submergence is ignored and the plate is assumed uniform.
Solutions to the Elastic Plate Model

- Model was proposed and an approximate solution was found by Wadhams 1986.
- First exact solution was by Fox and Squire 1994 for a 2d water-ice interface and by Meylan and Squire 1994 for 2d finite uniform floe.
- These results were extended to 3d floes of circular Meylan and Squire 1996 and arbitrary shape Meylan 2002.
- Very strong overlap with seaice research and research on Very Large Floating Structures.
Solution Method for 3d Arbitrary Floe

The problem is solved by converting Laplace’s equation to an integral equation

\[ \phi(x) = \phi_i(x) + \int \int_{\Delta} (G(x, y)\phi(y) - G(x, y)W(y)) \, dS \]

We solve this equation and the thin plate equation using a coupled BEM/FEM.
Connection between a single floe and scattering by thousands of floes is not clear.

Some kind of scaling or homogenization theory must apply but we do not know what it is.

Situation is very complicated, with wavelength and scatterer size similar, floes moving, random and complicated floe size, shape and thickness.

Various methods have been developed to connect single and multiple scattering and we briefly discuss these.
Boltzmann Equation Models

- Wadhams 1986 proposed a 2d model based on a transport equation.
- Masson and LeBlond 1989 and Meylan, Squire and Fox 1997 independently developed 3d Boltzmann/Transport models for wave scattering.
- These models were based on simplified floe solutions.
- These models turned out to be virtually identical.
- Equation describes the propagation of energy.

\[
\frac{1}{c_g} \frac{\partial I}{\partial t} + \theta \cdot \nabla I = -\beta I + \int_{0}^{2\pi} S(\theta, \theta') I(\theta') d\theta'
\]

- We do not know if the connection between equations in displacement and in energy are correct for random scattering.
Kohout and Meylan 2008 Model

- In some sense completed the 2d project proposed in Wadhams 1986.
- Simplified multiple scattering in 2d using the elastic plate model.
- Used extensive averaging of solutions for hundreds of random floes.
- Produced the first table of attenuation coefficients.
- Compared extensively to all available data.
- Good agreement except at long periods.
- Failed to correctly predict break up due to errors at long periods.
Modelling Wave Propagation, Scattering, and Attenuation in the Marginal Ice Zone

Figure: Break up
Modelling the MIZ

Figure: Break up
3d Array Models

- Models based on periodic arrays of floes have been developed by Peter and Meylan 2009 and Bennetts and Squire 2009.
- Quasi 3d using the solution for an infinite array of elastic plates with averaging over rows.
- Identical physics to the Kohout and Meylan model and results similar.
- Strange effects caused by the periodicity which we have not worked out how to average out.
Figure: Array Schematic
Figure: Array Results with and without Averaging
Other Notable Work

- Evans and Porter solved for scattering by cracks.
- Simplified water-ice problem solved by Wiener-Hopf and Residue calculus methods by Linton, Fox, Chung, et. al..
- Williams and Squire and Bennetts et. al. solved various problems with variable thickness.
- Almost all of this work has been two dimensional.
Measured Floe Thicknesses
Missing Physics

- Models do not correctly account for three dimensional effects.
- All non-linear effects are not included.
- Variable thickness is not correctly accounted for.
- Floe size and shape distribution is not included correctly.
Conclusions

- The problem of modelling wave scattering by the Marginal Ice Zone has been presented.
- Problem has many different aspects to its solution.
- Significant progress has been made but many obstacles need to be overcome to produce models suitable for making accurate predictions.