Channel Coordination under Competition
A Strategic View

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Newcastle
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Competition

Suppliers

Market
Bertrand model considers oligopolistic firms which compete with price.

Gallego and Hu [2007] consider the same setting, but firms are allowed to dynamically change their prices.
Coordination
A monopolistic supply chain is not coordinated with simple wholesale pricing contracts, due to several factors including:

- Double marginalization: Spengler [1950], ...
- Not enough sales effort: Cachon and Lariviere [2005], Taylor [2002], Krishnan et al. [2004], ...
Competition and Coordination

Suppliers -> Market

Suppliers

Retailer

Market
Elmaghraby [2000]: “If suppliers submit curves that reflect volume discount, then solving for the least-cost (set of) suppliers given the submitted curves is not guaranteed to be a simple task.”

Cachon [2003]: “More research is needed on how multiple suppliers compete for the affection of multiple retailers, i.e., additional emphasis is needed on many-to-one or many-to-many supply chain structures.”
- Contracts: general format quantity discounts
- Pricing: price taker or price setting retailer
- Method: non-cooperative non-zero-sum game theory
Definitions

\[ \pi(x) = \sum_i x_i p_i(x) - e(x) \]

- \( p_i(x) \): the price of product \( i \) when selling \( x \) units.
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Definitions

- $T_i$: the contract between supplier $i$ and the retailer.
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- $\pi(x|T) = \pi(x) - \sum T_i(x_i)$.
- $r(c|T) = \max\{\pi(x|T) : x \in [0, c]\}$.
- $s(c|T) = \text{arg max}\{\pi(x|T) : x \in [0, c]\}$.
**Assumption**

$r$ is submodular, i.e., has decreasing difference.

$(r(x \lor y) - r(x) \leq r(y) - r(x \land y))$. 

While supermodularity is preserved under maximization (Topkis [1998]), submodularity is not generally preserved under maximization. Maximizing a submodular function in general is NP hard.
Sub-modularity

Assumption

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- While supermodularity is preserved under maximization (Topkis [1998]), submodularity is not generally preserved under maximization.
- Maximizing a submodular function in general is NP hard.
Lemma

\( r(c) \) is submodular if:

- For all \( i = 1, \ldots, m \): \( p_i(x) \) is anti-multimodular,
- For all \( i = 1, \ldots, m \): \( p_i(x) \) is decreasing on \( x_j \) for all \( j = 1, \ldots, m \),
- \( e(x) \) is multimodular.

Multimodularity can be considered as a stronger assumption than submodularity.

Multimodularity results in component-wise concavity.

For Multimodularity definition and properties refer to Hajek [1985] and Li and Yu [2012].
Equilibrium

Theorem

In a competitive setting with sophisticated contracts, a set of contracts are Nash equilibrium if and only if they are in the following format:

\[
T_i^*(x) = \begin{cases} 
0 & \text{for } x = 0 \\
\geq r_i(c_{-i} + x) & \text{for } 0 < x < s_i(c) \\
r_i(c) & \text{for } s_i(c) \leq x \leq \min_{j\neq i} s_i(c_{-j}) \\
\geq r_i(c) & \text{for } \min_{j\neq i} s_i(c_{-j}) < x \leq c_i 
\end{cases}
\]

In addition, all Nash equilibria result in a coordinated chain with unique profit split as:

\[
T_i^*(s_i(c|T^*)) = r_i(c) = r(c) - r(c_{-i}) \\
r(c|T^*) = \sum_i r(c_{-i}) - (m-1)r(c)
\]
Proposition

A franchise contract is equilibrium if and only if it is in the following format:

\[ T_i^*(x) = r(c) - r(c_{-i}) \] for \( 0 < x \)
Proposition

A marginal units discount contract is equilibrium if and only if it charges price $p$ for units less than threshold $l$, and 0 for units over the threshold, where:

$$\frac{\partial r}{\partial c_i}(c_{-i}) \leq p \text{ and } l = \frac{r(c) - r(c_{-i})}{p}$$
Proposition

An all units discount contract is equilibrium if and only if it charges price $p$ if order is less than threshold $l$, and, $\frac{r(c) - r(c_i)}{s_i(c_i)}$ if order is more than or equal to threshold, where:

$\frac{\partial r}{\partial c_i}(c_{-i}) \leq p$ and $l = \min_{j \neq i} s_i(c_j)$
Example

- $m = 2$
- $c = [5, 10]$
- Fully substitutable products
- Market of size 10
- Fixed price of 20
- $\pi(x) = 20(x_1 + x_2) - (x_1 + x_2)^2 - 0.01x_1^2 - x_2$
Example: Contracts

![Graph showing the relationship between contracts and capacity for two suppliers. Supplier 1's contract growth is linear, while Supplier 2's is exponential, eventually reaching a plateau.](graph.png)
Comparative statics

Proposition

As a supplier’s capacity increases, her profit increases, the competitor suppliers’ profits decrease, and the retailer’s profit increases.
Example: Sensitivity to first supplier’s capacity

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Example: Sensitivity to second supplier’s capacity
Proposition

Under sophisticated contracts capacity buildings are coordinated, i.e., the equilibrium capacities are given by \( c^*_i(c^*_i) = \arg \max_{c_i} \{ r(c_i + c^*_i) \} \)
Wholesale Pricing

**Theorem**

In competitive setting with wholesale prices, there exists a Nash equilibrium, in which the chain is uncoordinated unless suppliers’ capacities are less than a threshold.
## Example: Win-lose analysis

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