. **Draft**

Spengler's Mathematics Considered or A Phoenix Reborn ?

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Introduction

Sometime during the first years of my graduate study, I came across Oswald Spengler's *The Decline of the West* (*Der Untergang des Abendlandes*, July, 1918). I surfed it a bit, considered it pretty heavy and indigestible cake, but made a mental note to look at it more carefully when I had the leisure. But I did notice at the time that Spengler had a chapter on mathematics quite at the beginning of Vol. I of his book: *The Meaning of Numbers* (*Vom Sinn der Zahlen*.)

More than a half century passed without my reading Spengler, but in the course of a recent group discussion of John Horgan's *The End of Science* and Fukuyama's *The End of History*, and Smolin's *The Trouble With Physics*, Spengler's name came up. Several of the participants had not heard of Spengler (!) and I determined then to take a look at *The Decline* -- at least the chapter on mathematics. What I found there was both intriguing and annoying, but I also found a number of notions with which I was in agreement and that therefore I was a Spenglerian in those regards. This discovery prompted the present article. After presenting some general information about Spengler and his book, I will concentrate on commenting on this one chapter. My views derive from my experience as a research mathematician with a lively interest in the history and sociology of mathematics

Spengler and *Der Untergang*

Oswald Spengler (1880 -1936) was he eldest of four children born to a middle class and conservative German family. His father was a white collar worker in the Post Office Department. As a child, Spengler read extensively, and later went to several universities reading in an undirected manner in many fields including mathematics and the natural sciences. Failing an appropriate degree, for a number of years he taught science, mathematics and German history in a number of high schools. In 1911, he came into a small inheritance and for the rest of his lonely life, living in Munich, he devoted himself to his writing. I do not believe that there is an in-depth English language biography of Spengler: there is probably very little to say of a personal nature. (But see both Hughes and Koktanek) On the other hand, with the appearance of *Der Untergang*, with its high sales, its critical approval and condemnation, its influence direct and indirect, positive and negative, on all sorts of thinkers, the amount of copy produced on Spengler's notions has been immense.

The Decline of the West is located among the apocalyptic works of which, over the centuries, there has been a steady production. The apocalyptic vision in the New Testament's *Book of Revelation* comes to mind. H.G.Wells' *The War of the Worlds* came out in 1898, and though Spengler was mainly influenced by Goethe and Nietzsche, he may very well have read Wells' vision of the future in translation.

American apocalyptic literature and art are all over the map. Henry Adams and his brother Brooks at the turn of the 20th Century wrote that the country and the world were going down the tube. Comparisons of the present state of things with the decline of the Roman Empire are so frequent that though troubling, they've become tedious. The production of post-apocalyptic fiction and movies seems endless. I would say that among academics the mood today is 80% Spenglerian (pessimistic) and 20% Macaulayan (optimistic), and I will explain the latter author later in this essay.

Now what, very briefly, does Spengler assert in *The Decline* ? History has patterns that are not linear but are cyclical. And just as an individual experiences birth, growth, maturity, decay, all great creations and

forms in art, religion, politics, economy, science, fulfill themselves and then die down . Man does not improve and contemporary man is on the downswing. Mathematics is no exception to the general rule of emergence and decay. At the very end of his chapter on number, he writes

> **"and with this culmination (the theory of groups) our western mathematic, having exhausted every inward possibility and fulfilled its destiny as *the copy and purest expression of the idea of the Faustian soul,* loses its development in the same way as the mathematic of the Classical Culture concluded in the third century. .... The time of the *great* mathematicians is past. Our tasks today are those of preserving, rounding off, refining, and selection -- in place of big dynamic creation. The same clever detail-work which characterized the Alexandrian mathematic of late Hellenism." p. 90.[1]**

Spengler distinguishes (admittedly in a fuzzy way) between Apollonian Man and Faustian Man. Apollonian Man is harmonious, measured, ordered, reasoned, limited, balanced in character[2]. Faustian Man strives beyond what is possible and to arrive at infinity. The classic Greek/Roman civilization was Apollonian, epitomized by statuary of great beauty; our Western civilization, beginning, say, in the 14th century is Faustian. Recall: in the Faust legend, a 16th Century tale of hubris and nemesis, Faust sold his soul to the Devil to acquire **all knowledge** (and one night with Helen of Troy.) . In between the Apollonian and the Faustian, Spengler interpolates the "Magian" man or culture. By this expression he means the culture associated with what he calls ‘Magian group of religions’ i.e., Judaism, ancient Chaldean religion, early Christianity, and Islam. As regards Magian mathematics, he means that of the Arabs (9th to the 15th Centuries.) For reasons which will emerge later, I find the adjective "Faustian" both appropriate and suggestive; I am indifferent to the terms "Apollonian" and "Magian."

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[1] Pages numbers refer to the translation by Atkinson. I maintain the italics and the capitalization. The word "mathematic" (sic) is used in the translation to indicate the mathematics of separate cultures. Occasionally I use this unusual term so as to connect with Spengler.

[2] The terms Apollonian and Dionysian derive from Nietzsche's *The Birth of Tragedy* where they designate two distinct aspects of Greek culture. The Dionysian is the intuitive, the orgiastic, etc. As regards classical mathematics, Spengler downplays the Dionysian.

The reception of *The Decline* was immediate and completely polarized. It became a best seller in whatever languages it was translated. Its advocates were mesmerized, stimulated, locked into its pessimism, and went on to write their own elaborations. Thus, Arnold J. Toynbee produced *A Study of History* in 12 volumes, describing the rise and fall of twenty two civilizations. More recently a whole spate of "gloom and doom" books have been published.

Spengler's detractors pointed to his factual mistakes, his lack of rational reasoning, his pompous posturing and prophesizing, his disturbing pessimism, his ignorance of this and that aspect of human history. Besides, they said that Spengler's main thesis was old hat. Predicting the future from the past, whether or not the forecasts are pessimistic or optimistic, is known in historiographic circles as "historicism." Generically, and apart from Spengler's variety of historicism, philosopher Karl Popper decried this methodology. Despite a strong polarization of opinion, one critic wrote that "It's easy to criticize Spengler. It's not so easy to get rid of him." In a letter of January, 1920, to Max Born, Einstein said that one goes to bed convinced of Spengler's ideas and wakes up aware of all that is wrong with them.

The Meaning of Numbers and its Reception

I come now to Spengler's Second Chapter in Volume I: *The Meaning of Numbers.*[3] Before discussing its ideas, I would like to point out that this chapter has been met with an almost absolute silence on the part of the Mathematical Establishment. You will not find a single reference to Spengler in the "standard" English language histories of mathematics. I have found a few short articles here and there. What explains this Great Silence, this *Todesstille,* as Spengler himself might have called it ?

I can come up with many reasons. The thought is that Spengler is simply wrong in his interpretation of the history of mathematics. He is uniformed and not up to date as to the fullness and richness of contemporary mathematical achievements. Einstein scoffed at Spengler's ignorance calling it *"Monomanie aus der Schullehrer-Mathematik..."* Yet,

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[3] Spengler's major discussion of mathematics occurs here. However there are less developed notices of mathematics later in Volume 1: e.g., destiny and the principle of causality in Chapter 3 , mathematical symbolisms and conceptions of space in Chapter 4.

writing in 1914 or so, Spengler takes note of Cantor's set theory which dates from the 1870's. Spengler expresses little interest in the applications of mathematics or how the physical world can elicit new mathematics. But he takes note of Minkowski's work on relativity which dates from just a few years before Spengler took pen in hand. He is weak on or ignores the mathematics of places like India, China, Central America or the Polynesian Islands.

More complaints: Spengler was a high school teacher and not a part of the mathematical establishment. He embedded his view of mathematics in a much wider and (often fuzzy) and problematic view of human history. He was not a Platonist and most practicing mathematicians are Platonists. His pessimism was pervasive, melodramatic and insufferable. None of us enjoy thinking of ourselves as living during the decaying end of a historic era. On the contrary, contemporary mathematicians think of the mathematical accomplishments of the past several centuries and especially of today, as alive and vibrant; a new efflorescence, full of new ideas and striking accomplishments and contributing to and formatting life as we now live it to a degree that the mathematicians of no previous era had accomplished. Consult Arnold et al. or Engquist and Schmid for many pages of such accomplishments. And yet, in an interview with Lennart Carleson, brilliant prize-winning mathematician and past president of the International Mathematical Union, reported in Engquist and Schmid, Carleson is quoted as saying "In all probability, we have been living in the golden age of mathematics." What next then ?

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There may also be political reasons for the silence. Spengler hated democracy; he saw the necessity (and even the advisability) of constant warfare. He backed the Nazis initially as a kind of fulfillment of his laws of development. But later he balked at its racial laws and his books were proscribed.

Yet, the silence was not absolutely total. One of the major English language notices of Spengler's chapter *The Meaning of Numbers* was from the pen of James R. Newman, an informed mathematical aficionado. Newman, a lawyer by training and practice, a social activist, a co-author of the Columbia mathematician Edward Kasner and of the logician Ernest Nagel. In 1956 his anthology *The World of Mathematics* appeared in four volumes. These volumes contained a collection of original articles by mathematicians together with brief introductions by Newman. Despite a

huge sale, these boxed volumes became "coffee table books," displayed but largely unread. After all, original mathematical papers are difficult; their study and contemplation lies in the domain of professional historians of mathematics; such works are not required reading for mathematics majors and graduate students and a fortiori are beyond the purview of the non-specialist.

In Volume 4 of Newman's anthology, he reprints the whole of Spengler's *The Meaning of Numbers*, together with his appraisal. Spengler's chapter, Newman writes, is

> **" one of the most remarkable discussions in *The Decline* . It is unnecessary to agree with Spengler's thesis to be stimulated by this performance. No one else has made even a comparable attempt to cast a synoptic eye over the evolving concept of number. A good deal of what Spengler has to say on this subject strikes one as far-fetched and misty.. [but] his ideas cannot be dismissed as hollow... This is a disturbing and exciting essay. "**

Historian and social activist, H. Stuart Hughes, writing a few years before Newman , considered this chapter as "perhaps the most provocative section of the whole *Decline*." and I agree: that it is impossible to dismiss it as unworthy of attention.

Clips from *The Meaning of Numbers*

It is now time for me to take up the contents of this "notorious" chapter which, in Atkinson's translation, runs to about fifty pages. The chapter is packed with everything that Spengler has ever thought about the history of mathematics and its relation to art, architecture, music, poetry, aesthetics, religious feelings and dogmas, mysticism, metaphysics. There is in it both redundancy and vagueness. In view of the length, density, and often impenetrability of the chapter, I can, alas, present only a few of Spengler's assertions -- ones that raised my eyebrows and occasionally my hackles.

Taking the concepts of number and of space and its interpretations as two basic indicators and characterizations of a civilization, Spengler compares and contrasts the number concept of the classical Greek/Roman period with that of the Western period. The former, as I have already noted,

he dubs Apollonian, the latter, Faustian. I shall clip and comment principally on Spengler's observations on the concept of number.

In the Classical period Spengler asserts that number relates to magnitude, while in the Western period it relates increasingly to relations.

While Pythagoras (c. 550 BCE) said that number is the essence of all things; Spengler modifies this to: the Classical formulation would be : "Number is the essence of all things *perceptible to the senses."* p. 63

The Classical number concept is limited in its scope and in its overtones:

> " Classical number is a thought-process dealing not with spatial relations but with visibly limitable and tangible units, and it follows naturally that the Classical knows only the 'natural' (positive and whole) numbers which on the contrary play in our Western mathematics a quite undistinguished part in the midst of complex, hypercomplex, non-Archimedean and other number systems...
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> **On this account, the idea of irrational numbers -- the unending decimal fractions of our notation -- was unrealizable within the Greek spirit. ... in considering the relation, say, between diagonal and side in a square, the Greek would be brought up suddenly a quite another sort of number , which was fundamentally alien to the Classical soul, and was consequently feared as a secret of its proper existence too dangerous to be unveiled." p. 64, 65.**

Among Classical numbers, there is no infinity, no infinitesimals, and no limiting processes.

> **"Classical mathematic of small things deals with the concrete *individual instance* and produces a once-for-all construction, while of mathematic of the infinite handles whole *classes* of formal possibilities, *groups* of functions, operations, equations, curves.."**

As regards numbers, they have become increasingly de-magnitude-ized:

> **"*the development of the new mathematic consists of a long, secret, and finally victorious battle against the notion of magnitude."* p.76**

"The history of Western knowledge is thus one of *progressive emancipation* from Classical thought..."

"And so for the last two centuries... there has been growing up the idea of a general morphology of mathematical operations, which we are justified in regarding as the real meaning of modern mathematics as a whole. p. 85

Considering the various features of Classical and Western mathematics:

"For the first time, it is possible to comprehend in full the elemental opposition of the Classical and Western souls. In the whole panorama of history, innumerable and intense as historical relations are, we find no two things so fundamentally alien to one another as these."

Apollonian man invented geometry, Magian man invented algebra, Faustian man invented the calculus and, with Descartes, geometry became increasingly separated from the visual experience. Spengler, who thought about art intensely, likes to make analogies that express his view of cultural parallelisms:

"The idea of Euclidean geometry is actualized in the earliest forms of Classical ornament, and that of infinitesimal calculus in the earliest forms of Gothic architecture, centuries before the first learned mathematicians of the respective Cultures were born. "

"The modern mathematic, though 'true' only for the Western spirit, is undeniably a master-work of that spirit; and yet to Plato it would have seemed a ridiculous and painful aberration from the path leading to the 'true' --to wit the Classical -- mathematic." p. 67

Predicting the twilight of Western mathematics, Spengler writes:

"Classical math was fulfilled by the end of the 2nd century AD."

"With this culmination, our Western mathematic having exhausted every inward possibility and fulfilled its destiny as the *copy and purest expression of the idea of the Faustian soul,* closes its development in the same way as the mathematic of the Classical Culture concluded in the third century." p.90

The divergence of spirit, of inner meaning, between the classical and the modern, leads Spengler to assert that

> **"*There is not and cannot be number as such.* There are several number-worlds as there are several Cultures. We find an Indian, an Arabian, a Classical, a Western type of mathematical thought and, corresponding with each, a type of number --each type fundamentally peculiar and unique, an expression of a specific world feeling, a symbol having a specific validity which is even capable of scientific definition, a principle of ordering the Become which reflects the central essence of one and only one soul , viz., the soul of that particular Culture.**
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> **Consequently there are more mathematics than one. For indubitably, the inner structure of Euclidean geometry is something quite different from that of the Cartesian , the analysis of Archimedes is something other than the analysis of Gauss, and not merely in matters of form, intuition and method, but above all in essence, in the intrinsic and obligatory meaning of number which they respectively develop and set forth. ... The style of any mathematic which comes into being, then, depends wholly on the Culture in which its is rooted, the sort of mankind it is that ponders it. The soul can bring its inherent possibilities to scientific development, can manage them practically, can attain the highest levels in its treatment of them -- but is quite impotent to alter them." p. 59.**

The incommensurability of the different cultures is one of Spengler's major points and thus he concludes,

> **"There are no eternal verities in this most abstract and disembodied intellectual activity."**

Personal reactions

Having given my readers a selection of quotes, I leave to the historians of science and mathematics whether the naked facts as asserted by Spengler are correct .For purposes of my reactions, I take them at face value. In the chapter under review, Spengler takes the long view in which a unit of time is perhaps 500 years. Contemporary mathematicians engaged primarily in teaching or in research, probably consider and need only the work of the past century, if that long a period. Their belief is that later work embraces

and incorporates past work even as it refines and strengthens it; and though we are often advised to "consider the works of the masters", we rarely do so. Furthermore, Spengler looks only at what he considers to be major tendencies and trends, perhaps a half dozen in all, often associated with the "great giants in the field." On the contrary, I would assert that there is an integrated corpus of mathematics that includes "the rounding off, the refinement," the further development, the reformulations and simplifications, the applications, without which the major trends, standing in isolation, would be unidentifiable and ultimately meaningless.

Numbers serve many functions. As cardinals they answer the question: how many. As ordinals they answer the question: how far along in a sequence. As tags they answer the question; which of many. Numbers are also the carriers of the magic, the metaphoric, the iconic.

The movement of number away from magnitude and into relations is clear. One cannot put the complex numbers or n-tuples of numbers into a linear ordering according to size. Numbers are seen now as instantiations of certain abstract structures obeying certain interrelationships. In the 1960's work of Grothendieck, the discrete (i.e., numbers) and the continuous (i.e., the mise en scène of the calculus) are combined into an arithmetic geometry, elevating the non-visual conception of space to a still higher level than that of linear algebra or Banach spaces.

It is very likely the case, that in an age of digital computer communication, by far the major use of number now occurs not to express magnitudes, but to transmit and then to decode and reinterpret trillions of long sequences of 0's and 1's as sentences in natural languages, as graphical images, or as some sort of mechanical or physical action.

The movement away from visualized space is clear. See the chapter *The Decline and Resurgence of the Visual in Mathematics* in Davis [7]. Does the current resurgence fit in with Spengler's theory of cycles ?

Spengler sees the mathematics of major cultures as being essentially autonomous; if mutual influences existed, they did not break the barrier of diverse inner expressions. He wishes to determine what those inner expressions are, to tell the story in it own terms without the intervention of contemporary knowledge and of our own contemporary spirit. In this regard, although he does not mention the name of the historian Otto von Ranke, he

is close to Ranke's famous instruction to write history "wie es eigentlich gewesen," to tell it in its own terms; how it really resided in and engaged the inner life of those who experienced it.

It is a very difficult task to get into the mind and spirit of an individual. It is, e.g., no easy matter to comprehend the original writings of Isaac Newton. One must not only master its Latin, its difficult geometric approach to what is now better expressed by differential equations. One must learn to appreciate how Newton's concern with Biblical exegesis enters the picture and to understand why John Maynard Keynes called Newton "the last of the magicians:"

> **"Newton was not the first of the age of reason. He was the last of the magicians, the last of the Babylonians and Sumerians, the last great mind which looked out on the visible and intellectual world with the same eyes as those who began to build our intellectual inheritance rather less than 10,000 years ago."**

I have said that Spengler's chapter is full of material which I have largely ignored in pulling out certain of his statements. The complete text points to a particular mind set which I can only partially understand. By looking hard at Spengler's life, his milieu, and his other writings, I might be able to get just a bit closer. Now if this is the case for one individual, how difficult it is to understand the mind-set of a whole civilization in the fullness of their individual lives.

Whig History of Mathematics

In opposition to this kind of (Rankean) historiography, there is what is now often called "Whig History." This term, exemplified early by Macaulay's *The History of England* (1849 - 1861), interprets history as a story of an inevitable progression toward the present. Whig history is optimistic, politically liberal, and materialistic. The past is prologue and things are getting better and better.

For the most part, comprehensive histories of mathematics are Whiggish. When a piece of ancient mathematics is explicated by translating its statements into current mathematical terminology and current symbols,

this makes for easier comprehension, but the historian is overlaying what is an ancient point of view with what is a contemporary interpretation in all its overtones.

While all history is selective, Whig history of mathematics, in its rational, Enlightenment point of view, has its own selectivity. As an example, it is well known that over the centuries, mathematics has had a special relation to theology. Whig history, in an act of intellectual cleansing to rid mathematics of what it considers as irrational, gives short shrift to the relations between theology and mathematics. There has been constant self-censoring to keep mathematics defined by what is considered to be its true and pure essence : axiomatization, deduction, proof, etc., applied to approved material.

Spengler's Introduction to Vol. 1 contains a sharp repudiation of Whig History:

> **"It is a quite indefensible method of presenting world-history to begin by giving rein to one's own religious, political or social conventions and endowing the sacrosanct three -phase system [child, youth, and man] with tendencies that will bring it exactly to one's own standpoint. This is, in effect, making of some formula.... a criterion whereby to judge the whole millennia of history. And so we judge that they [i.e., the past] were ignorant of the 'true path', or that they failed to follow it , when the fact is that their will and purposes were not the same as ours."**

Spengler's views are close to the recent emphasis and integrity given to "ethnomathematics" (in e.g., Ascher) a subject that has now taken on political colorations and is given short shrift within western-oriented histories. Today, mathematical expositors tend to view all these various "mathematics", taken together, as inter-interpretable, as a more or less linear and inevitable development to the present. The modern is regarded as the fulfillment of the classical Apollonian and the "Magian" just as in theological statements the *New Testament* is regarded as the fulfillment of the *Old Testament.* Spengler made no such claim; and those who assert dogmatically "the unity of mathematics" would scorn Spengler's denial.

Spengler's views are also close to what is now called "social constructivism" (see, e.g., Berger and Luckman) to which I am a partial

adherent believing that a good fraction of what is set forth as knowledge derives from and is validated by social interactions. As Paul Ernest put it,

> **"The social constructivist thesis is that mathematics is a social construction, a cultural product, fallible like any other branch of knowledge."**

A classic instance of social constructivism (or social parallelism) occurs in Paul Forman wherein he claims that the acausality embodied in quantum theory is directly related to and influenced by a general acausal intellectual *Weltanschauung* prevalent in the post WWI Weimar Republic of Germany. Forman shows how widely read Spengler was among Weimar mathematicians and physicists and how much their view of their fields was affected by the spirit and even by the letter of Spengler's book. This thesis was challenged by John Hendry with the counterclaim that acausality in quantum theory was determined largely by considerations internal to the physical evidence and the formulation of the associated mathematical theory. Both points of view have validity.

The End of Mathematics ?

Spengler was by no means the first to cry the "End of Mathematics." Consider the 1808 words of Jean-Baptiste Delambre (1749 -1822), distinguished mathematician and astronomer:

> **" It would be difficult and rash to analyze the chances which the the future offers to the advancement of mathematics; in almost all its branches one is blocked by insurmountable difficulties ; perfection of detail seems to be the only thing which remains to be done. All these difficulties appear to announce that the power of our analysis is practically exhausted."**

Consider also the views of the Scottish essayist and historian Thomas Carlyle. Carlyle (1795-1881) knew a bit of mathematics, taught it for a while, and in 1821 undertook to translate Legendre's *Eléments de Géométrie* which ran to many editions. But he soon ran into the stone wall of non-comprehension (which all of us do) and abandoned the field. In his 1829 essay *Signs of the Times* Carlyle wrote

> **"The science of the age is ... in all shapes mechanical. Our favourite Mathematics, the highly prized exponent of these other sciences, has also become more and more mechanical. Excellence in what is called its higher departments depends less on natural genius than on acquired expertness in wielding its machinery. Without undervaluing the wonderful results which a Lagrange or Laplace educes by means of it, we may remark that their calculus, differential and integral, is little else than a cunningly-constructed arithmetical mill; where the factors, being put in, are, as it were, ground into the true product, under cover, and without other effort on our part than steady turning of the handle. We have more Mathematics than ever; but less Mathesis. Archimedes and Plato could not have read the Mécanique Celeste: but neither would the whole French Institute see aught in that saying "God geometrises" but a sentimental rodomontade."**

The End of Mathematics ? How can that be? Current practitioners would be outraged at such a thought. Just consider all the developments, the new mathematical ideas that have been promulgated since Spengler wrote; the famous, long standing problems that have been solved, the work of the new crop of prize-winning mathematicians which is of the greatest depth and imagination; or consider the applications that now reach into every aspect of our lives and that promise an even wider reach. But how much more mathematics can there be ? Do we need more any more Bach or more Wordsworth or more Euler or Riemann ?

A Spenglerian " Mathematic" of the Future ?

In consonance with Spengler's assertion of cyclicity, we may view his thesis as an optimistic prediction of a new "mathematic", distinct from our present Faustian mathematics, that will arise. What are the hallmarks of the distinctions --admittedly problematic and disputed -- between the Apollonian, Magian, and Faustian or numerous other "ethnomathematics" ? They are distinct in virtue of great separation of time, space, and in the mindsets of the mathematicians as influenced by the conditions of life in their respective societies. Distinct mindsets lend more than formal interpretations to the lines of naked symbols. In view of modern communications that have greatly reduced both space and time, and computers that have enlarged our so-called intelligence and relieved the

burden of formal manipulation, might we anticipate a new Spenglerian mathematic to arise in the not too distant future: the Phoenix reborn from its Faustian ashes ?

What harbingers of such a rebirth can I detect ? A review of the major theories or trends of the 20th Century yield, among numerous others, logic/set theory, Bourbakism, catastrophe theory, wavelets, category theory, fractals. Of these, I think that fractals, born of the computer and with its initial deduction-free outlook, comes closest to presaging a new mathematic. Ignoring the embarrassing question of whether the mathematics of the past century or so poses nothing essentially new -- "having [in Delambre and Spengler's sense] exhausted every inward possibility," let us instead concentrate on how mathematics is now changing.

The change is rapid, profound, still totally Faustian to use Spengler's term. The computer is changing the mode of discovery, operation and conceptualization of mathematics. It has overcome some of the "insurmountable difficulties" mentioned by Delambre. It is changing notions of what mathematics is interesting and even what mathematics is. It is my belief that the old paradigms and emphases, especially the current stress on proof, will become dated and as long as they last, will be considered as exercises in ritualistic nostalgia.

The philosopher Alfred North Whitehead remarked somewhere that all later philosophy consists of nibbling on the rump of Plato. Western mathematics has all along, but more substantially since the early 1800's, been nibbling on the rump of Euclid. Some years back, [7] I suggested the possibility of accepting into the mathematical canon "visual theorems" generated with an assist from the computer.

> **[A computer generated visual image] "is a gestalt, complete in itself, self-vindicating, rejoicing in its uniqueness, the carrier for an unlimited number of `theorems of perceived type' that are grasped or intuited and *do not even have to be stated.*"**

This raised only a few eyebrows.

Jonathan Borwein with parallel but deeper views, emphasizes the role of the experimental mathematics via the computer, and presents a substantial outline of a possible new mathematic. Borwein predicts (or at least

advocates) a mathematics that emphasizes induction, empirical discipline , and finding rather than proving. He writes:

> **" I hope to have made convincing arguments that the traditional deductive accounting of Mathematics is a largely ahistorical caricature -- Euclid's millennial sway not withstanding,."**

Paradoxically, the numerous interesting and supporting examples that Borwein and I have brought forth in support are firmly lodged in what might be called neo- Eulerian mathematics.(I refer here to the derivation of numerous special function identities and not to Euler's position on proof which was favorable.) Thus, this kind of development would not of itself be a mathematic of the future in the Spenglerian sense.

I recently had occasion to read an article in the field of Computer AI (Artificial Intelligence.) I found that I was immediately thrust into a different mathematical world than the one I grew up in. I found different modes of exposition, different goals, different kinds of argumentation, different criteria of validity, and a different clientèle. And yet, the influence of my generation of mathematics is clear: axiomatization, logics, topological considerations, etc. Despite this "new world " feeling, is it enough to constitute a new mathematic in the Spenglerian sense ? I think not. Such a presumptive mathematic should fit in with and reflect a new social *Zeitgeist* (which I cannot yet detect) ; it should constitute a threat to what is now around, should raise the question as to what mathematics really is, and raise the cry among Old Believers that it is not mathematics at all.

Mathematics and Society

Mathematics underlies our whole civilization in a way that would have astounded, pleased, but confused Pythagoras. Build a bridge, conduct an election, study the galaxies and in some way you will engage mathematics. Make an investment and mathematics plays a role. Consider DNA profiling and you enter the burgeoning fields of mathematical genetics and bioinformatics.

The world is being mathematized, computerized, chipified at an increasing rate and the public is hardly aware that mathematics is its basis.

Mathematics is a method and a language employed in increasing amounts to give order and to format our social, economic and political lives. It is a method and an attitude that has diffused into medicine, cognitive science, war, entertainment, art, aesthetics, law, sports; an attitude that has created schools of philosophy, and has given support to views of cosmology, mysticism, and theology.

I combine the high marks I give to mathematics with a measure of skeptical caution. I believe that mathematics and its applications lie between common sense and the irrelevance of common sense, between what is intuitive and what is counter intuitive, between the obvious and the esoteric. Mathematics creates the infinitely large and the infinitely small. It sets forth ideals that cannot be achieved by human actions.

Judging from the articles and books about the future produced at the turn of the millennium (2000), e.g., in Engquist and Schmid, the sky's the limit with little thought for an Icarian nemesis. Every technological innovation has its down side. This is the message of the Myth of Prometheus who stole fire from the gods and gave it to the humans. Mathematics and its applications have a downside and Spengler was aware of this. Mathematics frequently transforms subjective opinions into so-called objective conclusions bearing the cachet of absolute truth. In the name of logic, mathematics can create seeming impossibilities and nonsense. It the name of social ideals, conceived as logical truths, it can persuade and seduce individuals (thankfully, rarely) to commit anti-social acts.

Natural languages are symbolic systems that have raised humans from the level of Caliban brutes, and the same is true of mathematics. It is a language that has transformed our lives for good but it can go hog wild when it becomes the handmaiden of new and unprecedented dimensions of human cruelty. The ethical issues in science are in the daily papers. The ethical issues involved in mathematical thinking should also be recognized and pondered so as -- in the words of Bertrand Russell -- "to tip the balance on the side of hope against vast forces."

We are, as asserted by Spengler, in the Faustian Age and this will persist for the foreseeable future. The spirit of Googolization now wafts over all enterprises, and like that of Faust, wants to embrace all, capture the whole of human culture in one vast database provided with search engines

and other peripherals. Giant businesses are swallowed up by behemothian businesses; theories of everything (TOE's) are promulgated in physics; and in mathematics, generalizations, abstractions and "foundations" are sought that purport to wrap up more and more with less and less. Even Hilbert's program of proposing to formalize **all** of mathematics, of discovering a method for deciding that truth of **any** mathematical statement, may be viewed as Faustian. (Of course this program crashed on the rocks of Gödel's Incompleteness Theorem.) More than Faustian: if one considers and gives credence to the predictions emerging from the futuristic think tanks,one would conclude that the computer is moving us inevitably and with juggernaut force towards a super- or trans- Faustian civilization. Thus, Hank Bostrom, the Director of the Future of Humanity Institute at Oxford, in a scenario reminiscent of sci-fi, envisions the creation of computers with more brain power than all the humans in the world put together, and the consequent emergence of "post-humans" who play around with virtual ancestors, etc., and where the distinction between the real and the virtual has totally disappeared.

But "The End of Mathematics ? " The language of mathematics is inextricably embedded in natural languages and hence a vital part of communication. Can one imagine an end to language ? But whether or not the languages of the future will bring forth expressions that will recall and match the Homers, the Archimedes, the Shakespeares, the Newtons, or the Riemanns is beyond our vision but not our hope.

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References

1. V. Arnold, M. Atiyah, P. Lax, and B. Mazur eds., *Mathematics: Frontiers and Perspectives*, Amer. Math. Soc, 1999.

2. Marcia Ascher, *Ethnomathematics: A Multicultural View of Mathematical Ideas*, Brooks/Cole, 1991

3. Peter L. Berger and Thomas Luckman, *The Social Construction of Reality*, Anchor, 1966

4. Jonathan Borwein, *Implications of Experimental Mathematics for the Philosophy of Mathematics*, (also in Gold and Simons [15])
http://users.cs.dal.ca/~jborwein/implications.pdf

5. Nick Bostrom, Philosophical Quarterly, 2003, Vol. 53, No. 211, pp. 243-255.

6. C. Chant and J. Fauvel, eds, *Darwin to Einstein: Historical Studies on Science and Belief*, Longmans, 1980.

7. Philip J. Davis, *Visual Geometry, Computer Graphics and Theorems of Perceived Type*, Proceedings of Symposia in Applied Mathematics, vol. 20, American Mathematical Society, 1974.

8. Philip J. Davis, *Mathematics and Common Sense: A Case of Creative Tension.* AK Peters, 2006

9. Björn Engquist and Wilfried Schmid, eds, *Mathematics Unlimited,-- 2001 and Beyond*, Springer, 2001.

9. Paul Ernest, *Social Construction as a Philosophy of Mathematics*, SUNY Press, 1998.

11. John Farrenkopf *Prophet of Decline: Spengler on World History and Politics*.. Baton Rouge: Louisiana State University Press. 2001.

12. John F. Fennelly, *The Twilight of the Evening Lands*, Brookdale Press, 1972, esp. pp 144-155.

13. Paul Forman, *Weimar Culture, Causality and Quantum Theory*, Historical Studies in the Physical Sciences, Vol. 3, pp 1-115. 1971. An abridged version can be found in Chant and Fauvel. Also on line: http://ohst.berkeley.edu/events/Conferences/FormanConference/index.html.

14. Francis Fukuyama , *The end of history and the last man,* Free Press , 1992

15. Bonnie Gold and Roger Simons eds., *Contemporary Thoughts About Mathematics: Philosophy of Mathematics, for Mathematicians* . To appear.

16. John Hendry, *Weimar Culture and Quantum Causality*, in Chant and Fauvel.

17. John Horgan , *The End of Science : Facing the Limits of Knowledge in the Twilight of the Scientific Age* , Addison-Wesley ,1996

18. H. Stuart Hughes: *Oswald Spengler*, Scribner's, 1952, 1992 Edition: Transaction Publishers.

19. Morris Kline: *Mathematical Thought from Ancient to Modern Times*, Oxford, 1972. p. 623.

20. Anton M. Koktanek, *Oswald Spengler in seiner Zeit*, C.H. Beck, 1968

21. James R. Newman, *The World of Mathematics* , Vol. 4, Simon and Schuster, 1956

22. Sal Restivo, *The Social Relations Of Physics, Mysticism, And Mathematics***,** D. Reidel, , 1983.

23. Lee Smolin, *The Trouble with Physics*, Houghton Mifflin, 2006.

24. Oswald Spengler, *Der Untergang des Abendlandes*, C.H. Beck. 1918-1922.

24. Oswald Spengler, *The Decline of the West,* C.F. Atkinson translator, Knopf, 1926