Locally pro-$p$ contraction groups are nilpotent

George A. Willis & H. Glöckner

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Abstract

A contraction group is a pair $(G, \alpha)$ in which $G$ is a locally compact group and $\alpha$ is an automorphism of $G$ such that $\alpha^n(x) \to 1$ as $n \to \infty$.

In joint work with H. Glöckner, it is shown that every contraction group is the direct sum of closed subgroups $G = D \oplus T$ with $D$ divisible (i.e. for every $x \in D$ and $n > 0$ there is $y \in D$ with $y^n = x$) and $T$ torsion (i.e., there is $n > 0$ such that $x^n = 1$ for every $x \in T$). Furthermore, $D$ is the direct sum $D = \bigoplus_{i=1}^{k} D_{p_i}$ of $p_i$-adic analytic nilpotent contraction groups for some prime numbers $p_1, \ldots, p_k$.

The torsion subgroup $T$ may also be written as a composition series of simple contraction groups. In the case when all the composition factors are of the form $(\mathbb{F}_p((t)), \alpha)$ with $\alpha$ being the automorphism of multiplication by $p$, it follows easily that $G$ is a solvable group. These ideas will be explained in the talk and a sketch will be presented of a proof that $G$ is in fact nilpotent in this case.